To multiply a polynomial by a monomial.

Distributing a Monomial

Factoring Out a Monomial

... And Why

To factor a monomial out of a polynomial, as in Example 3.

What You’ll Learn

- To multiply a polynomial by a monomial
- To factor a monomial from a polynomial

Check Skills You’ll Need

Multiply.
1. 3(302) 906
2. 41(7) 287
3. 9(504) 4536

Simplify each expression.
4. 4(6 + 5x) 24 + 20x
5. −8(2y + 1) −16y − 8
6. (5v − 1)5 25v − 5
7. 7(p − 2) 7p − 14
8. (6 − x)9 54 − 9x
9. −2(4x − 1) −8x + 2

In Chapter 2 you used the Distributive Property to multiply a number by a sum or difference.

5(a + 3) = 5a + 15 (x − 2)(3) = 3x − 6 −2(2y + 7) = −4y − 14

You can also use the Distributive Property or an area model to multiply polynomials.
Consider the product 2x(3x + 1).

2x(3x + 1) = 2x(3x) + 2x(1)
= 6x² + 2x

You can use the Distributive Property for multiplying powers with the same base when multiplying by a monomial.

Example

Multiplying a Monomial and a Trinomial

Multiple Choice

Simplify

- You can use the Distributive Property. Multiply the coefficients and add the exponents of powers with the same base.
- Simplify.

So A is the correct answer.

Quick Check

1. Simplify each product.
   a. 4(5b² + b² + 6) 20b³ + 4b² + 24b
   b. −7h(3h² − 8h − 1) −21h³ + 56h² + 7h
   c. 2(x² − 6x + 5) 2x³ − 12x² + 10x
Factoring a Monomial From a Polynomial

Factoring a polynomial reverses the multiplication process. To factor a monomial from a polynomial, first find the greatest common factor (GCF) of its terms.

**Example 2** Finding the Greatest Common Factor

Find the GCF of the terms of $4x^3 + 12x^2 - 8x$.

List the prime factors of each term. Identify the factors common to all terms.

$4x^3 = 2 \cdot 2 \cdot x \cdot x \cdot x$

$12x^2 = 2 \cdot 2 \cdot 3 \cdot x \cdot x$

$8x = 2 \cdot 2 \cdot 2 \cdot x$

The GCF is $2 \cdot 2 \cdot x$ or $4x$.

**Quick Check**

Find the GCF of the terms of each polynomial.

a. $5v^3 + 10v^3$  

b. $3t^2 - 18$  

c. $4b^3 - 2b^2 - 6b$  

To factor a polynomial completely, you must factor until there are no common factors other than 1.

**Example 3** Factoring Out a Monomial

Factor $3x^3 - 12x^2 + 15x$.

**Step 1** Find the GCF.

$3x^3 = 3 \cdot x \cdot x \cdot x$

$12x^2 = 2 \cdot 2 \cdot 3 \cdot x \cdot x$

$15x = 3 \cdot 5 \cdot x$

The GCF is $3 \cdot x$ or $3x$.

**Quick Check**

Use the GCF to factor each polynomial.

a. $8x^2 - 12x$  

b. $5d^3 + 10d$  

c. $6m^3 - 12m^2 - 24m$

**EXERCISES**

For more exercises, see Extra Skill and Word Problem Practice.

Simplify each product. 5–12. See back of book.

1. $8m(m + 6)$  

2. $(x + 10)3x$  

3. $9k(7k + 4)$  

4. $-5a(a - 1)$  

5. $2x^2(9 + x)$  

6. $-p^3(p - 11)$  

7. $2x(x^3 - x^2 + 5x)$  

8. $4y^2(9y^3 + 8y^2 - 11)$  

9. $-5c^3(9c^2 - 8c - 5)$  

10. $-7q^2(6q^2 - 2q - 7)$  

11. $-3g^2(6g^2 - 6g + 5)$  

12. $-x^6(10x^3 + 3x^2 - 7)$

Find the GCF of the terms of each polynomial.

13. $15w + 21$  

14. $6a^2 - 8a$  

15. $36v + 24$  

16. $x^3 + 7x^2 - 5x$  

17. $5b^3 + 15b - 30$  

18. $9x^3 - 6x^2 + 12x$

**Lesson 9-2** Multiplying and Factoring 501

**Additional Examples**

2. Find the GCF of the terms of $2x^4 + 10x^2 - 6x$.  

2x

3. Factor $4x^3 - 8x^2 + 12x$.  

$4x(x^2 - 2x + 3)$

**Closure**

Ask students how to find the GCF of a polynomial. First, list the prime factors of each coefficient. Then identify the factors common to all coefficients. Finally, write the product of these common factors with the least power of the variable used in any term.

**Advanced Learners**

Help students understand that factoring and multiplying binomials and polynomials are each applications of the distributive property.

Learning style: verbal

**English Language Learners**

In Exercise 42, be sure students understand the meaning of diagonal and vertex and vertices by having them identify these terms on the diagram.

Learning style: visual
### Building Models

Suppose you are building a model of the square castle shown at the left. The moat of the model castle is made of blue paper. Do the moat of your model castle have more area than the moat shown in the diagram? Why or why not?

#### Math Tip

Factored Form

Suppose you are building a model of the square castle shown at the left. The moat of the model castle is made of blue paper. Do the moat of your model castle have more area than the moat shown in the diagram? Why or why not?

#### Exercise 33

European castles usually consisted of two concentric square or triangular shaped systems of steep walls and round towers surrounding an inner courtyard. Outside the outer wall, a deep dry ditch or a moat served as defense, garbage dump, and sewer. Moats also contained eels and fish for food.

#### Exercise 44

Remind students that “in terms of s” means that s is the variable in the formula.

### Challenge 43

#### Geometry

How many sides does the polygon have? How many of its diagonals come from one vertex? 6; 3

Suppose a polygon has n sides. How many diagonals will it have from one vertex? n – 3

The number of diagonals from all the vertices is \( \frac{n}{2} (n - 3) \). Multiply the two factors. \( \frac{n}{2} \cdot \frac{n}{2} = \frac{n^2}{4} \)

For a polygon with 8 sides, what is the total number of diagonals that can be drawn from the vertices? 20
44. **Manufacturing**  The diagram shows a cube of metal with a cylinder cut out of it. The formula for the volume of a cylinder is $V = \pi r^2 h$, where $r$ is the radius and $h$ is the height.

a. Write a formula for the volume of the cube in terms of $s$. $V = 64s^3$

b. Write a formula for the volume of the cylinder in terms of $s$. $V = 48\pi s^2$

c. Write a formula for the volume of the metal left after the cylinder has been removed. $V = 64s^3 - 48\pi s^2$

d. Factor your formula from part (c). $V = 16s(4s - 3s)$

e. Find $V$ in cubic inches for $s = 15$ in. **about 182,071 in.$^3$**

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**Test Prep**

**Multiple Choice**

45. $x(6x^2 - 4x - 2)$ equals which of the following expressions? **B**

A. $6x^3 - 4x - 2$  
B. $6x^2 - 4x^2 - 2x$  
C. $6x^3 - 4x^2 - 2$  
D. $7x^3 - 5x^2 - 3x$

46. Simplify $(p^2 - 3) - (5 - p + 2p^2) - (4p + 5 - 2p^2)$. What is the coefficient of $p^2$? **F**

F. 1  
G. 2  
H. 4  
J. 5

47. Let $[n]$ represent the number of different pairs of integers whose product is $n$. For example, $-1 \times 10$, $-2 \times 5$, $1 \times (-10)$, and $2 \times (-5)$ give $-10$. So $[10] = 4$. What does $[24]$ equal? **C**

A. 4  
B. 6  
C. 8  
D. 10

48. Which of the following represents an odd number for any integer $n$? **G**

F. $n + 1$  
G. $2n + 1$  
H. $3n$  
J. $3n + 1$

49. Factor $6g^6 - 3g^4 + 9g^2$ completely. **B**

A. $g^2(6g^4 - 3g^2 + 9)$  
B. $3g^2(2g^6 - g^2 + 3)$  
C. $g^2(6g^6 - 3g^2 + 9g)$  
D. $3g^2(2g^6 - g^2 + 9g)$

**Short Response**

50. How do you know if you've factored out the GCF of a polynomial? Illustrate your explanation by using the GCF to factor $10x^3 + 6x^2 + 2x^2$.

See margin.

**Mixed Review**

**Lesson 9-1**

Simplify. Write each answer in standard form. 52–54. See margin.

51. $(x^2 + 3) - (4x^2 - 7) - 3x^2 + 10$  
52. $(m^3 + 8m + 6) + (-5m^3 + 4m)$

53. $(g + 6g - 2) + (4g + 7g + 2)$  
54. $(3r^2 - 8r + 7) - (2r^2 + 8r - 9)$

55. $(t^4 + t^3 + 1) + (3 + 5t^2 - 10)$  
56. $(3b^3 - b^2) - (5b^2 + 12)$

57. $t^4 + 5t^2 - 9$  
363 - 6b^2 - 12

**Lesson 8-1**

Simplify each expression.

57. $\frac{5-1}{5}$  
58. $\frac{-2}{25}$

59. $(\sqrt{2})^{-3}$  
60. $\sqrt{8}$

61. $n^{-3}n^2$  
62. $(2w^3)^2$

63. $\frac{1}{c} + 4c^3$  
64. $\frac{b^2 - 8}{3}$

**Lesson 7-3**

Solve by elimination.

65. $7x + 6y = 33$  
66. $8x + 4y = 28$

2x - 6y = -6 (3, 2)  
3x - 2y = 21 (5, -3)

67. $4x + 2y = 16 (1, 6)$  
11x - 3y = -7

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**Multiple Choice**

50. **[2]** $2x^2(5x^2 + 3x + 1);$ the terms $5x^2, 3x,$ and $1$ have no common factor other than $1$.

51. **[1]** incorrect factoring OR incorrect explanation

52. $m^2 - 5m^2 + 12m + 6$

53. $5g^2 - g$

54. $r^2 - 16r + 16$

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**Alternative Assessment**

Organize students into groups of three. Using the examples in this lesson, have each group write a similar polynomial and its factors on cards. For example, students should write $6x^2 - 4x$ on one card, the factor $2x$ on one card, and the factor $(3x - 2)$ on a third card. Collect all the cards from the class and shuffle them. Randomly redistribute the cards to the class. Have students find their matching cards. Tell students not to be alarmed if there is more than one card with the same factor on it. Each factor will have a polynomial to match it.

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**Test Prep**

**Resources**

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 545
- Test-Taking Strategies, p. 540
- Test-Taking Strategies with Transparencies

**Exercise 48** Remind students that any odd number is one more than an even number. $2n + 1$ always represents an even number for any integer $n.$