**Solving Systems Using Substitution**

**What You’ll Learn**
- To solve systems using substitution

... And Why
To solve problems involving transportation, as in Example 3

**Check Skills You’ll Need**
Solve each equation.
1. \( m - 6 = 4m + 8 - 4 \)
2. \( 4n = 9 - 2n \)
3. \( \frac{1}{2}t + 5 = 10 \)

For each system, is the ordered pair a solution of both equations?
4. \((5, 1)\) \( y = -x + 4 \)
\( y = x - 6 \) no
5. \((2, 2.4)\) \( 4x + 5y = 20 \)
\( 2x + 6y = 10 \) no

**New Vocabulary**
- substitution method

**Using Substitution**

**Vocabulary Tip**
Substitution means one value or expression is used in place of another.

You can solve a system of equations by graphing when the solution contains integers or when you have a graphing calculator. Another method for solving systems of equations is the substitution method. By replacing one variable with an equivalent expression containing the other variable, you can create a one-variable equation that you can solve using methods shown in Chapter 2.

**Example**

Using Substitution

Solve using substitution. \( y = -4x + 8 \)
\( y = x + 7 \)

**Step 1** Write an equation containing only one variable, and solve it.
\( y = -4x + 8 \) Start with one equation.
\( x + 7 = -4x + 8 \) Substitute \( x + 7 \) for \( y \).
\( 5x + 7 = 8 \) Add \( 4x \) to each side.
\( 5x = 1 \) Subtract 7 from each side.
\( x = 0.2 \) Divide each side by 5.

**Step 2** Solve for the other variable in either equation.
\( y = 0.2 + 7 \) Substitute 0.2 for \( x \) in \( y = x + 7 \).
\( y = 7.2 \) Simplify.

Since \( x = 0.2 \) and \( y = 7.2 \), the solution is \((0.2, 7.2)\).

**Check**
\( 7.2 \neq -4(0.2) + 8 \) Since \( y = x + 7 \) was used in step 2, see if \((0.2, 7.2)\) solves \( y = -4x + 8 \).
\( 7.2 = 7.2 \) Simplify.

**Quick Check**
Solve using substitution. Check your solution. \( y = 2x \)
\( 7x - y = 15 \) \((3, 6)\)

**Differentiated Instruction**

**Special Needs**
If some students have difficulty distinguishing the colors of two different lines on a graph, redraw the lines in two different thicknesses or have them work with a partner.

**Below Level**
Remind students that they must check that a solution makes all equations in the system true in order for it to be a solution of the entire system.

**learning style: visual**

**learning style: verbal**
To use the substitution method, you must have an equation that has already been solved for one of the variables.

**EXAMPLE**  Using Substitution and the Distributive Property

Solve using the substitution method. 

\[ 3y + 2x = 4 \]
\[ -6x + y = -7 \]

**Step 1** Solve the second equation for \( y \) because it has a coefficient of 1.

\[ y = 6x - 7 \quad \text{Add 6x to each side.} \]

**Step 2** Write an equation containing only one variable and solve.

\[ 3y + 2x = 4 \quad \text{Start with the other equation.} \]
\[ 3(6x - 7) + 2x = 4 \quad \text{Substitute 6x - 7 for y. Use parentheses.} \]
\[ 18x - 21 + 2x = 4 \quad \text{Use the Distributive Property.} \]
\[ 20x = 25 \quad \text{Combine like terms and add 21 to each side.} \]
\[ x = 1.25 \quad \text{Divide each side by 20.} \]

**Step 3** Solve for the other variable in either equation.

\[ -6(1.25) + y = -7 \quad \text{Substitute 1.25 for x in } -6x + y = -7. \]
\[ -7.5 + y = -7 \quad \text{Simplify.} \]
\[ y = 0.5 \quad \text{Add 7.5 to each side.} \]

Since \( x = 1.25 \) and \( y = 0.5 \), the solution is \((1.25, 0.5)\).

**Quick Check**

Solve using substitution. Check your solution.

\[ 6y + 8x = 28 \]
\[ 3 = 2x - y \]

\((2.3, 1.6)\)

**EXAMPLE**  Real-World Problem Solving

**Transportation**  Your school is planning to bring 193 people to a competition at another school. There are eight drivers available and two types of vehicles, school buses and minivans. The school buses seat 51 people each, and the minivans seat 8 people each. How many buses and minivans will be needed?

Let \( b \) = number of school buses. Let \( m \) = number of minivans.

- \( b + m = 8 \) people
- \( 51b + 8m = 193 \)

**Step 1** \( b + m = 8 \) Solve the first equation for \( m \).

\[ m = -b + 8 \quad \text{Subtract b from each side.} \]

**Step 2** \( 51b + 8(-b + 8) = 193 \) Substitute \(-b + 8\) for \( m \) in the second equation. \( 51b - 8b + 64 = 193 \)

\[ 43b + 64 = 193 \]
\[ 43b = 129 \]
\[ b = 3 \]

**Step 3** \( b + m = 8 \) Substitute 3 for \( b \) in \( b + m = 8 \).

\[ m = 5 \]

Three school buses and five minivans will be needed to transport 193 people.

**Geometry**  A rectangle is 4 times longer than it is wide. The perimeter of the rectangle is 30 cm. Find the dimensions of the rectangle. \(3 \text{ cm by } 12 \text{ cm}\)

**Guided Instruction**

**Error Prevention**

Some students may equate \( 51b \) with 51 buses. Point out that \( b \) represents the number of buses, and 51 represents the number of people each bus can hold.

**Additional Examples**

1. Solve using substitution.
\[ y = 2x + 2 \]
\[ y = -3x + 4 \]
\[ (0.4, 2.8) \]

2. Solve using substitution.
\[ -2x + y = -1 \]
\[ 4x + 2y = 12 \]
\[ (1.75, 2.5) \]

3. A youth group with 26 members is going to the beach. There will also be five chaperones that will each drive a van or a car. Each van seats 7 persons, including the driver. Each car seats 5 persons, including the driver. How many vans and cars will be needed?

3 vans and 2 cars

**Resources**

- Daily Notetaking Guide 7-2
- Daily Notetaking Guide 7-2—Adapted Instruction

**Closure**

Ask: Why is it sometimes easier to solve equations using substitution rather than graphing? Sometimes the solution is a very large number or a decimal.
3. Practice

Assignment Guide

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Homework Quick Check

To check students’ understanding of key skills and concepts, go over Exercises 6, 18, 22, 24, 32.

Exercises 1–4: Suggest to students that they first substitute the coordinate into the equation in each system that is easiest to work with, such as \( x = y \) in Exercise 3.

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EXERCISES

Practice and Problem Solving

**Mental Math** Match each system with its solution at the right.

1. \( y = x + 1 \) **D**
   \( y = 2x - 1 \)
2. \( y = \frac{1}{2}x + 4 \) **C**
   \( y + 2x = 2 \)
3. \( 2y = x + 3 \) **B**
   \( x = \frac{1}{2}y + 2 \)
4. \( x - y = 1 \) **A**
   \( x = 2y \)

Solve each system using substitution. Check your solution. 5–16. See margin.

5. \( y = 4x - 8 \)
   \( y = 2x + 10 \)
   \( 6. \ C(n) = -3n - 6 \)
   \( C(n) = n - 4 \)
   \( 8. \ y = -4x + 12 \frac{1}{2} \)
   \( y = \frac{1}{3}x + 4 \)
   \( 9. \ h = 6g - 4 \)
   \( h = -2g + 28 \)
   \( 10. \ a = \frac{2}{5}b - 3 \)
   \( a = 2b - 18 \)
   \( 11. \ y = x - 2 \)
   \( 2x + 2y = 4 \)
   \( 12. \ c = 3d - 27 \)
   \( 4d + 10c = 120 \)
   \( 13. \ 3x - 6y = 30 \)
   \( y = -6x + 34 \)
   \( 14. \ m = 4n + 11 \)
   \( -6n + 8m = 36 \)
   \( 15. \ 7x - 8y = 112 \)
   \( y = -2x + 9 \)
   \( 16. \ t = 0.2x + 10 \)
   \( 4x + 5t = 35 \)

**Example 1** (page 382)

**GO for Help**

**Example 2** (page 383)

**Example 3** (page 383)

17. **Geometry** The length of a rectangle is 5 cm more than twice the width. The perimeter of the rectangle is 34 cm. Find the dimensions of the rectangle.

18. Suppose you have $28.00 in your bank account and start saving $18.25 every week. You purchase 8 gal of paint and 3 brushes for $152.50. The next day, you purchase 6 gal of paint and 2 brushes for $113.00. How much does each gallon of paint and each brush cost?

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Apply Your Skills

Solve each system by substitution. Check your solution.

19. \( a - 1.2b = -3 \)
   \( 0.2b + 0.6a = 12 \) **(15, 15)**
   \( 0.5x + 0.25y = 36 \)
   \( y + 18 = 16x \) **(9, 126)**
   \( 20x + 32y = 48 \) **(-4, 4)**

For Exercises 22–24, define variables and write a system of equations for each situation. Solve using substitution.

22. **Agriculture** A farmer grows only sunflowers and flax on his 240-acre farm. This year he wants to plant 80 more acres of sunflowers than of flax. How many acres of each crop does the farmer need to plant?

23. **Renting Videos** Suppose you want to join a video store. Big Video offers a special discount card that costs $9.99 for one year. With the discount card, each video rental costs $2.49. A discount card from Main Street Video costs $20.49 for one year. With the Main Street Video discount card, each video rental costs $1.79. After how many video rentals is the cost the same? **15 video rentals**

24. **Buying a Car** Suppose you are thinking about buying one of two cars. Car A will cost $17,655. You can expect to pay an average of $1230 per year for fuel, maintenance, and repairs. Car B will cost about $15,900. Fuel, maintenance, and repairs for it will average about $1425 per year. After how many years are the total costs for the cars the same? **9 yr**

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Real-World Connection

Sunflower seeds are sold as snacks and as bird food, and they are a source of cooking oil.
25. **Multiple Choice** You have 28 coins that are all nickels $n$ and dimes $d$. The value of the coins is $2.05. Which system of equations can be used to find the number of nickels and the number of dimes? D

- $n + d = 28$
- $10n + 5d = 2.05$
- $10n + 5d = 205$
- $n + d = 205$
- $n + d = 28$
- $5n + 10d = 205$

**Estimation** Graph each system to estimate the solution. Then use substitution to find the exact solution of the system. 26–31. See back of book.

26. $y = 2x$
   - $y = -6x + 4$
   - $y = -4x - 5$
   - $5x + 2y = -3$
27. $y = \frac{1}{2}x + 4$
   - $y = 2x + 6$
   - $y = -1.5x - 7$
28. $x + y = 0$
   - $x = 2x$
   - $y = 2x$
29. $y = 2x + 3$
   - $y = -x + 4$
   - $31. y = 0.7x + 3$
30. $y = -x + 4$
   - $y = 2x + 6$
   - $y = 0.5x - 2$
32. **Open-Ended** Write a system of linear equations with exactly one solution. Use substitution to solve your system. See left.

**Challenge**

- 32. Answers may vary. Sample: $y = x$ and $y = -3x + 2; (\frac{2}{3}, \frac{1}{2})$

33. a. Solve each system below using substitution. $(x, y)$ such that $y = 0.5x + 4$; no solution

   - $y = 0.5x + 4$
   - $6x - 2y = 10$
   - $x + 2y = 8$
   - $y = 3x + 1$

b. Solve each system by graphing. See back of book.

c. **Critical Thinking** Make a general statement about the solutions you get when solving by graphing and the results you get when solving by substitution. See margin.

**Solve each system using substitution.**

34. $y = 2x$
   - $6x - y = 8$
   - $(2, 4)$
35. $y = 3x + 1$
   - $x = 3y + 1$
   - $(-\frac{1}{2}, -\frac{1}{2})$
   - $x = 2 = 0$
   - $(2, -4)$
36. $x - 3y = 14$
   - $-2x - 3y = 1$
   - $(-\frac{1}{2}, 0)$
   - $x + 4y = -4$
   - $(4, -2)$

37. $2x + 2y = 5$
   - $y = \frac{1}{2}x$
   - $(2, \frac{1}{2})$
38. $4x + y = -2$
   - $-2x + 3y = 1$
39. $3x + 5y = 2$
   - $x + 4y = -4$

40. There are 1170 students in a school. The ratio of girls to boys is 23 : 22.

   The system below describes relationships between the number of girls and the number of boys.

   - $g + b = 1170$
   - $\frac{g}{b} = \frac{23}{22}$

   a. Solve the proportion for $g$. $g = \frac{23}{22}b$
   b. Solve the system. $(b, g) = (572, 598)$
   c. How many more girls are there than boys? 26

41. **Sprinting** The graph at the left represents the start of a 100-meter race between Joetta and Gail. The red line and blue line represent Joetta’s and Gail’s time and distance. Joetta averages 8.8 m/s. Gail averages 9 m/s but started 0.2 s after Joetta. At time 0.2, Gail’s distance is 0 m. You can use point-slope form to write an equation that relates Gail’s time $t$ to her distance $d$.

   $y - y_1 = m(x - x_1)$
   
   $d - 0 = 9(t - 0.2)$
   
   $d = 9t - 1.8$

   Since Joetta started at $t = 0$, the equation $d = 8.8t$ relates her time and distance.

   a. Solve the system using substitution. $(t, d) = (9, 79.2)$
   b. Will Gail overtake Joetta before the finish line? yes

**Error Prevention!**

**Exercise 17** Some students may use $p = 34$ for the second equation. Explain to students that the expression for finding perimeter, $2l + 2w$, must be used so that the equations have the same variables.

**Alternative Method**

**Exercise 18** Students may want to solve this problem by keeping two running balances like those used in bank account registers. Have them make side-by-side vertical lists to find when the numbers are the same.

<table>
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<th>Week</th>
<th>$28 + 18.25$</th>
<th>$161 - 15$</th>
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<tbody>
<tr>
<td>1</td>
<td>46.25</td>
<td>146.00</td>
</tr>
<tr>
<td>2</td>
<td>64.50</td>
<td>131.00</td>
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<td>etc.</td>
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<td>101.00</td>
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</tbody>
</table>

46. **Challenge**

47. **Challenge**

48. **Challenge**

49. **Challenge**

50. **Challenge**

33. **Open-Ended** When there is exactly one solution, the graph will show two lines that intersect in a single point, and the substitution method will give the coordinates of the point. When there are infinitely many solutions, the graphs coincide, and substitution leads to an equation that is always true. When there are no solutions, the graphs are parallel, and substitution results in an equation that is never true.

45. [2] $7(-7) \neq 4(-2) \neq 29$
   - $-49 + 8 \neq 29$
   - $-49 \neq 29$

No, $(-2, -7)$ must satisfy both equations to be a solution of the system.

[1] no explanation given
Find the value of the $y$-coordinate of the solution to the given system. $5x + 5y = 179$ $x = 5y - 143$ 29.8

Find the value of the $y$-coordinate of the solution to the given system. $y = 9x + 3480$ $y = 81x - 7104$ 4803

Tina has $220 in her account. Cliff has $100 in his account. Starting in July, Tina adds $25 to her account on the first of each month, while Cliff adds $35 to his. How many dollars will they have in their accounts when the amounts are the same? 520

Lessons 7-1 through 7-2

Systems of Equations and Inequalities

A farmer grows only pumpkins and corn on her 420-acre farm. This year she wants to plant 250 more acres of corn than of pumpkins. How many acres of each crop does the farmer need to plant? $250$ acres corn, 335 acres pumpkins

Checkpoint Quiz 1

1. $y = 3x - 4$ $y = -2x + 1 (1, -1)$
   $y = x - 2$ $y = \frac{1}{4}x - 2 (3, 2)$
   $y = \frac{1}{4}x - 1 (-4, -2)$

2. $y = 3x - 14$ $y = 2x + 5 (2, -8)$
   $y = 6x + 1 (1, 7)$
   $y = -2x - 10 (-4, -2)$

3. $3x + 4y = 12$ $4x + 9y = 24$
   $y = -2x + 10 \left(\frac{28}{5}, \frac{6}{5}\right)$ $y = -\frac{1}{2}x + 2 (6, 0)$

In Exercises 9 and 10, write and solve a system of equations for each situation.

9. A rectangle is 3 times longer than it is wide. The perimeter is 44 cm. Find the dimensions of the rectangle. $2L + 2W = 44, 3W = L; 5.5$ cm by 16.5 cm

10. A farmer grows only pumpkins and corn on her 420-acre farm. This year she wants to plant 250 more acres of corn than of pumpkins. How many acres of each crop does the farmer need to plant? $250$ acres corn, 335 acres pumpkins

Chapter 7 Systems of Equations and Inequalities