Solving Multi-Step Equations

What You’ll Learn

• To use the Distributive Property when combining like terms
• To use the Distributive Property when solving equations

... And Why
To solve a problem involving building a fence, as in Example 2

Check Skills You’ll Need

1. $2n - 3n = n$
2. $-4 + 3b + 2 + 5b = 8b - 2$
3. $9(w - 5) = 9w - 45$
4. $-10(b - 12) = -10b + 120$
5. $3(-x + 4) = -3x + 12$
6. $5(6 - w) = 30 - 5w$

Evaluate each expression.
7. $28 - a + 4a$ for $a = 5$
8. $8 + x - 7x$ for $x = -3$
9. $(8n + 1)3$ for $n = -2$
10. $-(17 + 3y)$ for $y = 6 - 3$

Using the Distributive Property to Combine Like Terms

You can solve equations that require more than two steps. If there are like terms on one side of an equation, first use the Distributive Property to combine them. Then use the properties of equality to solve the equation.

1. Combining Like Terms

Problem Solving Hint

You use the Distributive Property whenever you add or subtract like terms.

\[
x + 4x = 1x + 4x = (1 + 4)x = 5x
\]

EXAMPLE

Combining Like Terms

Solve each equation.

a. $2c + c + 12 = 78$
   
   $2c + c + 12 = 78$

   $3c + 12 = 78$

   $3c + 12 - 12 = 78 - 12$

   $3c = 66$

   $\frac{3c}{3} = \frac{66}{3}$

   $c = 22$

b. $4b + 16 + 2b = 46$
   
   $4b + 16 + 2b = 46$

   $4b + 2b + 16 = 46$

   $6b + 16 = 46$

   $6b + 16 - 16 = 46 - 16$

   $6b = 30$

   $\frac{6b}{6} = \frac{30}{6}$

   $b = 5$

Check Skills You’ll Need

For intervention, direct students to:

Exponents and Order of Operations

Lesson 1-2: Example 2
Extra Skills and Word Problem Practice, Ch. 1

The Distributive Property

Lesson 2-4: Examples 3, 4
Extra Skills and Word Problem Practice, Ch. 2
You can model real-world situations using multi-step equations.

**Real-World Problem Solving**

**Multiple Choice** A gardener is planning a rectangular garden area in a community garden. His garden will be next to an existing 12-ft fence. The gardener has a total of 44 ft of fencing to build the other three sides of his garden. How long will the garden be if the width is 12 ft?

- **A** 32 ft
- **B** 24 ft
- **C** 22 ft
- **D** 16 ft

**Relate** length of side plus 12 ft plus length of side equals amount of fencing.

**Define** Let \( x \) = length of a side adjacent to the fence.

**Write**

\[
\begin{align*}
\frac{2}{2}x + 12 &= 44 \\
\frac{2}{2}x &= 32 \\
x &= 16
\end{align*}
\]

The garden will be 16 ft long. So D is the correct answer.

**Quick Check**

A carpenter is building a rectangular fence for a playground. One side of the playground is the wall of a building 70 ft wide. He plans to use 340 ft of fencing material. What is the length of the playground if the width is 70 ft?

\[ 135 \text{ ft} \]

**Using the Distributive Property to Solve Equations**

In the equation \(-2(b - 4) = 12\), the parentheses indicate multiplication. Use the Distributive Property to multiply each term within the parentheses by \(-2\). Then use the properties of equality to solve the equation.

**Example**

Solve \(-2(b - 4) = 12\).

\[
\begin{align*}
-2b + 8 &= 12 \\
-2b &= 4 \\
b &= -2
\end{align*}
\]

**Quick Check**

Solve each equation.

a. \(3(k + 8) = 21\) \( \boxed{-1} \)

b. \(15 = -3(x - 1) + 9\) \( \boxed{-1} \)

**Guided Instruction**

**1. Tactile Learners**

Let students model and solve the equation with tiles to help them understand that \(2c + c = 3c\).

**2. Auditory Learners**

Have pairs of students take turns explaining the steps of this example to each other. Then, have them work together to solve the Check Understanding exercises.

**Advanced Learners**

Challenge students to solve the equation in Example 5 by a different method than the one shown.

**English Language Learners**

Some students may not know what a community garden (referred to in Example 2) is. Let a student who has seen or participated in a community garden explain what one is and how it is run.
Following are two ways you can solve an equation like $\frac{3x}{2} + \frac{x}{2} = 7$.

### Vocabulary Tip

$\frac{x}{2}$ and $\frac{1}{2}x$ both represent $x \div 2$. $\frac{2x}{3}$ and $\frac{x}{3}$ both represent $2x \div 3$.

### Example

#### Solving an Equation That Contains Fractions

**Method 1** Adding fractions

\[
\frac{2x}{3} + \frac{x}{2} = 7
\]

Rewrite the equation with fractions as coefficients.

\[
\frac{2x}{3} + \frac{x}{2} = 7
\]

Write the fractions with a denominator of 6.

\[
\frac{4x}{6} + \frac{3x}{6} = 7
\]

Combine like terms.

\[
\left(\frac{7}{6}\right) x = 7
\]

Multiply each side by $\frac{6}{7}$, the reciprocal of $\frac{7}{6}$.

\[
x = 6
\]

Simplify.

#### Method 2** Multiplying to clear fractions

\[
\frac{2x}{3} + \frac{x}{2} = 7
\]

Multiply each side by 6, a common multiple of 3 and 2.

\[
6\left(\frac{2x}{3} + \frac{x}{2}\right) = 6(7)
\]

Use the Distributive Property.

\[
4x + 3x = 42
\]

Multiply.

\[
7x = 42
\]

Combine like terms.

\[
\frac{7x}{7} = \frac{42}{7}
\]

Divide each side by 7.

\[
x = 6
\]

Simplify.

### Quick Check

Solve each equation. Explain why you chose the method you used.

**a.** $\frac{3x}{4} + \frac{2x}{2} = \frac{5}{8}$

**b.** $\frac{5}{3}x - \frac{5}{8}x = 26$

*a-b. Explanations may vary.*

You can clear an equation of decimals by multiplying by a power of 10. In the equation $0.5a + 8.75 = 13.25$, the greatest number of digits to the right of a decimal point is 2. To clear the equation of decimals, multiply each side of the equation by $10^2$, or 100.

### Example

#### Solving an Equation That Contains Decimals

Solve $0.5a + 8.75 = 13.25$.

\[
100(0.5a + 8.75) = 100(13.25)
\]

Multiply each side by $10^2$, or 100.

\[
100(0.5a) + 100(8.75) = 100(13.25)
\]

Use the Distributive Property.

\[
50a + 875 = 1325
\]

Simplify.

\[
50a + 875 - 875 = 1325 - 875
\]

Subtract 875 from each side.

\[
50a = 450
\]

Simplify.

\[
\frac{50a}{50} = \frac{450}{50}
\]

Divide each side by 50.

\[
a = 9
\]

Simplify.

### Quick Check

Solve each equation.

**a.** $0.025x + 22.95 = 23.65$  

**b.** $1.2x - 3.6 + 0.3x = 2.4$
Keep the steps in the summary below in mind as you solve equations that have variables on one side of the equation.

### Key Concepts

**Steps for Solving a Multi-Step Equation**

- **Step 1**: Clear the equation of fractions and decimals.
- **Step 2**: Use the Distributive Property to remove parentheses on each side.
- **Step 3**: Combine like terms on each side.
- **Step 4**: Undo addition or subtraction.
- **Step 5**: Undo multiplication or division.

### EXERCISES

For more exercises, see Extra Skill and Word Problem Practice.

#### Practice and Problem Solving

**Practice by Example**

**Example 1** (page 126)

Solve each equation. Check your answer.

1. $4n - 2n = 18$
2. $y + y + 2 = 18$
3. $a + 6a - 9 = 30$
4. $5 - x - x = -1$
5. $72 + 4 - 14c = 36$
6. $13 = 5 - 13 + 3a$
7. $9 = -3 + n + 2n$
8. $7m - 3m - 6 = 6$
9. $-13 = 2b - b - 10$

**Example 2** (page 127)

Write an equation to model each situation. Solve your equation.

10. Two friends are renting an apartment. They pay the landlord the first month’s rent. The landlord also requires them to pay an additional half of a month’s rent for a security deposit. The total amount they pay the landlord before moving in is $1725. What is the monthly rent? $x + \frac{1}{2}x = 1725; \$1150$

11. You are fencing a rectangular puppy kennel with 25 ft of fence. The side of the kennel against your house does not need a fence. This side is 9 ft long. Find the dimensions of the kennel. $x + 9 + x = 25; 8 \text{ ft by 9 ft}$

**Example 3** (page 127)

Solve each equation. Check your answer.

12. $2(8 + p) = 22$
13. $5(a - 1) = 35$
14. $15 = -3(2q - 1)$
15. $26 = 6(5 - a)$
16. $m + 5(m - 1) = 7$
17. $-4(x + 6) = -40$
18. $48 = 8(x + 2)$
19. $5(y - 3) = 19$
20. $5(2 + y) = 77$

**Example 4** (page 128)

21. $\frac{4}{3} - \frac{5}{6} = \frac{7}{11}$
22. $x - \frac{5}{8} = \frac{7}{12}$
23. $\frac{mn}{6} - 7 = \frac{2}{3}$
24. $\frac{3}{2} + \frac{3k}{4} = \frac{7}{12}$
25. $4 + \frac{m}{8} = \frac{3}{4}$
26. $\frac{4}{2} + \frac{1}{5} = 17$
27. $\frac{1}{2} + \frac{7v}{10} = \frac{13}{20}$
28. $\frac{9y}{14} + \frac{3}{7} = \frac{9}{10}$
29. $\frac{1}{3} + \frac{3w}{15} = \frac{4}{5}$

**Example 5** (page 128)

30. $3m + 4.5m = 15$
31. $7.8y + 2 = 165.8$
32. $3.5 = 12x - 5x$
33. $1.06y - 3 = 0.71$
34. $0.11p + 1.5 = 2.49$
35. $25.24 = 5y + 3.89$
36. $1.12 + 1.25y = 8.62$
37. $1.025x + 2.458 = 7.583$
38. $0.25m + 0.1m = 9.8$
Solve each equation.

39. \(0.5t - 3t + 5 = 0\)
40. \(- (z + 5) = -14\)
41. \(\frac{a}{5} + \frac{4}{15} = \frac{9}{15}\)
42. \(0.5(x - 12) = 4\)
43. \(8y - (2y - 3) = 9\)
44. \(\frac{1}{2} + y = \frac{3}{4} \cdot 12\)
45. \(2 + \frac{a}{4} = \frac{3}{5} \cdot \frac{3}{5}\)
46. \(\frac{3}{4}(m - 16) = 7\)
47. \(x + 3x - 7 = 29\)
48. \(4x + 3.6 + x = 1.2\)
49. \(2(1.5c + 4) = -1 \cdot 3\)
50. \(26.54 - p = 0.5(50 - p)\)

51. **Error Analysis** Explain the error in the student’s work at the right. **The student forgot to multiply \(-1\) by 8.**

52. **Critical Thinking** Suppose you want to solve the equation \(-3m + 4 + 5m = -6\). What would you do as your first step? **See left.**

53. **Writing** To solve \(-\frac{1}{2}(3x - 5) = 7\), you can use the Distributive Property, or you can multiply each side of the equation by \(-2\). Which method do you prefer? Explain why. **Answers may vary. Sample: Multiply by \(-2\) to eliminate fractions.**

54. **Geometry** The perimeter of each rectangle is 64 in. Find the value of \(x\).

55. **Use an equation to solve each problem.**

56. **Canoe Rental**

<table>
<thead>
<tr>
<th>CANOE RENTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.00 per hour</td>
</tr>
<tr>
<td>$2.00 life jacket</td>
</tr>
</tbody>
</table>

John and two friends rent a canoe at a park. Each person must rent a life jacket. If the bill for the rental of the canoe and life jackets is $41, for how many hours did they rent the canoe? **7 h**

57. **Moving Costs** The MacNeills rented a moving truck for $49.95 plus $.30 per mile. Before returning the truck, they filled the tank with gasoline, which cost $18.32. The total cost was $95.87. Find the number of miles the truck was driven. **92 mi**

58. **Cell Phones** Jane’s cell phone plan is $40 per month plus $.15 per minute for each minute over 200 minutes of call time. If Jane’s cell phone bill is $58.00, for how many extra calling minutes was she billed? **120 min**

59. **Open-Ended** Write an expression with four terms that can be simplified to an expression with two terms. **Answers may vary. Sample: 3x + 5 - 4x + 9**

60. **Geometry** Find the value of \(x\). (Hint: The sum of the measures of the angles of a triangle is \(180^\circ\).)
Lesson Quiz

1. \(4a + 3 - a = 24\)
2. \(-3(x - 5) = 66 - 17\)
3. \(12 = 7\)
4. \(0.05x + 24.65 = 27.5\)

Alternative Assessment

Write \(0.4(m - 2) - 0.2m = -0.2\). Have students solve the equation showing all work. Instruct them to write the number of the Step from p. 91 that is used for each step of the solution.

Test Prep

Resources

For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 195
- Test-Taking Strategies, p. 190
- Test-Taking Strategies with Transparencies

Test Prep

Multiple Choice

69. What is the value of the expression \(-3r + 6 + r\) when \(r = -2\)?
   A. \(-6\)   B. \(-2\)   C. 10   D. 14

70. Solve \(8n + 5 - 2n = 41\).
   F. \(3\frac{1}{2}\)   G. \(4\frac{1}{2}\)   H. 6   J. \(7\frac{1}{2}\)

71. If a number is increased by 3 and that number is doubled, the result is \(-8\).
   What was the original number?
   A. \(-7\)   B. \(-5.5\)   C. 1   D. 6
72. The gas tank in Royston's car holds 12 gal of gasoline. The car averages 29 mi/gal. Royston filled up the tank and then drove 140 mi. About how many gallons of gasoline are left in the tank?  
   F. 6 gal    G. 7 gal    H. 8 gal    J. 9 gal

73. Josie's goal is to run 40 miles each week. This week she has already run distances of 5.3 miles, 6.5 miles, and 6.2 miles. If she wants to spread out the remaining miles evenly over the next 4 days, which equation can you use to find how many miles \( m \) per day she must run?  
   A. \( 5.3 + 6.5 + 6.2 + 40 = m \)  
   B. \( 40 - 5.2 - 6.5 - 6.2 = m \)  
   C. \( 5.3 + 6.5 + 6.2 + 4m = 40 \)  
   D. \( 5.3 + 6.5 + 6.2 + m = 40 \)

74. A cell phone company charges $.35 for the first minute but only $.10 every minute after that. Which equation can you use to find how many minutes \( m \) Eric talked if the bill for the call was $5.45?  
   F. \( 0.35 + 0.10(m - 1) = 5.45 \)  
   G. \( 0.35 + 0.10m = 5.45 \)  
   H. \( 0.10 + 0.35(m - 1) = 5.45 \)  
   J. \( 0.10 + 0.35m = 5.45 \)

### Multiplication Property of Equality

- If \( a = b \), then \( ca = cb \).  
- If \( a = b \), then \( \frac{a}{c} = \frac{b}{c} \), for all \( c \neq 0 \).

### Dividing Both Sides

- If \( \frac{a}{c} = \frac{b}{c} \), then \( a = b \).  
- If \( a = b \), then \( \frac{a}{c} = \frac{b}{c} \), for all \( c \neq 0 \).

### Algebra at Work

#### Airline Pilot

Airline pilots make many calculations before, during, and after a flight. Pilots study weather conditions to determine the safest altitude, route, and speed for a flight. Pilots calculate lift, which must equal the airplane’s weight in pounds. An airplane’s lift capabilities are calculated using the formula \( L = \frac{1}{2} d v^2 s a \), where \( L \) is the lift, \( d \) is the density of the air, \( v \) is the velocity of the aircraft in feet per second, \( s \) is the wing area of the aircraft in square feet, and \( a \) is a value determined by the type of airfoil the airplane has and the pitch angle of the airplane.