5-2 Relations and Functions

What You'll Learn

- To identify relations and functions
- To evaluate functions

. . . And Why

To determine whether a relation is a function, as in Examples 1 and 2

Check Skills You'll Need

GO for Help Review page 24 and Lesson 1-2

Graph each point on a coordinate plane. 1–4. See back of book.

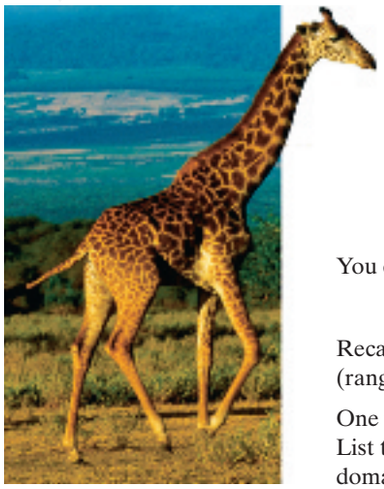
1. $(2, -4)$ **2.** $(0, 3)$ **3.** $(-1, -2)$ **4.** $(-3, 0)$

Evaluate each expression.

5. $3a - 2$ for $a = -5$ **−17** **6.** $9(x - 9)$ for $x = 3$ **−54** **7.** $3x^2$ for $x = 6$ **108**

New Vocabulary • relation • vertical-line test • function notation

1 Identifying Relations and Functions



A **relation** is a set of ordered pairs. The (age, height) ordered pairs below form a relation.

Giraffes

Age (years)	18	20	21	14	18
Height (meters)	4.25	4.40	5.25	5.00	4.85

You can list the set of ordered pairs in a relation using braces.

$$\{(18, 4.25), (20, 4.40), (21, 5.25), (14, 5.00), (18, 4.85)\}$$

Recall from Lesson 1-4 that a function is a relation that assigns exactly one output (range) value for each input (domain) value.

One way you can tell if a relation is a function is by making a *mapping diagram*. List the domain values and the range values in order. Draw arrows from the domain values to their range values.

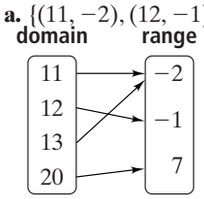
1 EXAMPLE Using a Mapping Diagram

Online active math

For: Function Activity
Use: Interactive Textbook, 5-2

Determine whether each relation is a function.

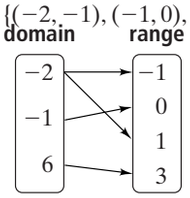
a. $\{(11, -2), (12, -1), (13, -2), (20, 7)\}$



There is no value in the domain that corresponds to more than one value of the range.

The relation is a function.

b. $\{(-2, -1), (-1, 0), (6, 3), (-2, 1)\}$



The domain value corresponds to two range values, −1 and 1.

The relation is not a function.

Quick Check 1 Use a mapping diagram to determine whether each relation is a function.

a. $\{(3, -2), (8, 1), (9, 2), (3, 3), (-4, 0)\}$ **not a function**

b. $\{(6.5, 0), (7, -1), (6, 2), (2, 6), (5, -1)\}$ **function**

Differentiated Instruction Solutions for All Learners

Special Needs L1
Have students discuss the meaning of a machine in terms of what fuels a machine and what the machine produces. Ask students for examples that they can show or draw, and explain to the class.

learning style: verbal

Below Level L2
Ask students to determine if the data in the two tables on p. 241 represent functions. Have them use the vertical-line test for one set of data and a mapping diagram for the other.

learning style: visual

5-2

1. Plan

Objectives

1 To identify relations and functions
2 To evaluate functions

Examples

1 Using a Mapping Diagram
2 Using the Vertical-Line Test
3 Making a Table From a Function Rule
4 Find the Range

Professional Development

Math Background

Functions and relations do not need to involve numbers. For example, the relationship that assigns a color to each pixel on a computer screen is a function.

More Math Background: p. 250C

Lesson Planning and Resources

See p. 250E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:

Graphing Data on the Coordinate Plane
Review p. 24: Example 2

Exponents and Order of Operations
Lesson 1-2: Example 2
Extra Skills and Word Problem Practice, Ch. 1

2. Teach

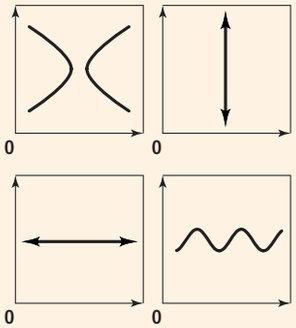
Guided Instruction

1 EXAMPLE

List all the *x*-values in Question a. Ask: *Do any x-values repeat?* **no** Repeat for Question b. **yes** Stress to students that if an *x*-value repeats and has a different *y*-value, there will be more than one point on the vertical line passing through the *x*-value. This violates the vertical-line test. Have students sketch each set of points so they can see the alignment of the points.

2 EXAMPLE Teaching Tip

Sketch the following graphs on the board or overhead projector. Then let student volunteers perform the vertical line test using a pencil or a yardstick.

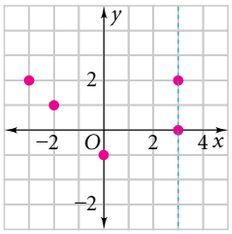


Another way you can tell whether a relation is a function is to analyze the graph of the relation using the **vertical-line test.** If any vertical line passes through more than one point of the graph then for some value of x there is more than one value of y. Therefore, the relation is not a function.

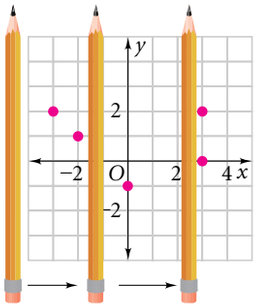
2 EXAMPLE Using the Vertical-Line Test

Determine whether the relation $\{(3, 0), (-2, 1), (0, -1), (-3, 2), (3, 2)\}$ is a function.

Step 1 Graph the ordered pairs on a coordinate plane.



Step 2 Pass a pencil across the graph as shown.



A vertical line would pass through (3, 0) and (3, 2). The relation is not a function.

Quick Check 2 Use the vertical-line test to determine whether each relation is a function.

a. $\{(4, -2), (1, 2), (0, 1), (-2, 2)\}$ **function**

b. $\{(0, 2), (1, -1), (-1, 4), (0, -3), (2, 1)\}$ **not a function**

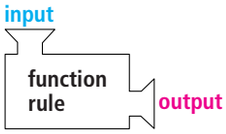
2 Evaluating Functions

Recall from Lesson 1-4 that a function rule is an equation that describes a function. You can think of a function rule as an input-output machine.

Words and Notations Used With a Function

Domain	Range
input	output
x	$f(x)$
x	y

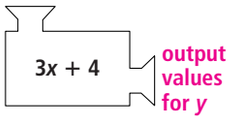
The domain is the set of input values.



The range is the set of output values.

If you know the input values, you can use a function rule to find the output values. The output values depend on the input values.

$$\underset{\text{output}}{y} = \underset{\text{input}}{3x} + 4$$



Input	Output
x	y
1	7
2	10
3	13

Another way to write the function $y = 3x + 4$ is $f(x) = 3x + 4$. A function is in **function notation** when you use $f(x)$ to indicate the outputs. You read $f(x)$ as "f of x" or "f is a function of x." The notations $g(x)$ and $h(x)$ also indicate functions of x.

In Lesson 1-4 you wrote function rules from tables. You can also make a table using values from a function rule.

Differentiated Instruction Solutions for All Learners

Advanced Learners L4
Challenge students to find the meaning of $g(f(x))$.
learning style: verbal

English Language Learners ELL
Some students may not understand the Vertical-Line Test in Example 2. Have them do Steps 1 and 2. Explain that if there are 2 or more y-values for one *x*-value, the Vertical-Line Test fails, so the relation is not a function.
learning style: tactile

3 EXAMPLE Making a Table From a Function Rule

Vocabulary Tip

You can think of the notation $f(6)$ as "Replace n with 6 to find the value of $f(6)$."

Make a table for $f(n) = -2n^2 + 7$. Use 1, 2, 3, and 4 as domain values.

n	$-2n^2 + 7$	$f(n)$
1	$-2(1)^2 + 7$	5
2	$-2(2)^2 + 7$	-1
3	$-2(3)^2 + 7$	-11
4	$-2(4)^2 + 7$	-25

Quick Check 3 Make a table for $y = 8 - 3x$. Use 1, 2, 3, and 4 as domain values. **See back of book.**

You can use a function rule and a given domain to find the range of the function. After computing the range values, write the values in order from least to greatest.

4 EXAMPLE Finding the Range

Evaluate the function rule $f(a) = -3a + 5$ to find the range of the function for the domain $\{-3, 1, 4\}$.

$f(a) = -3a + 5$ $\quad$ $f(a) = -3a + 5$ $\quad$ $f(a) = -3a + 5$

$f(-3) = -3(-3) + 5$ $\quad$ $f(1) = -3(1) + 5$ $\quad$ $f(4) = -3(4) + 5$

$f(-3) = 14$ $\quad$ $f(1) = 2$ $\quad$ $f(4) = -7$

The range is $\{-7, 2, 14\}$.

Quick Check 4 Find the range of each function for the domain $\{-2, 0, 5\}$.

a. $f(x) = x - 6$ **$\{-8, -6, -1\}$**

b. $y = -4x$ **$\{-20, 0, 8\}$**

c. $g(t) = t^2 + 1$ **$\{1, 5, 26\}$**

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice.*

Practice and Problem Solving

A Practice by Example

GO for Help

Example 1 (page 257)

Use a mapping diagram to determine whether each relation is a function.

1. $\{(3, 7), (3, 8), (3, -2), (3, 4), (3, 1)\}$ **no**
2. $\{(6, -7), (5, -8), (1, 4), (5, 5)\}$ **no**
3. $\{(0.04, 0.2), (0.2, 1), (1, 5), (5, 25)\}$ **yes**
4. $\{(4, 2), (1, 1), (0, 0), (1, -1), (4, -2)\}$ **no**

Example 2 (page 258)

Use the vertical-line test to determine whether each relation is a function.

5. $\{(2, 5), (3, -5), (4, 5), (5, -5)\}$ **yes**
6. $\{(5, 0), (0, 5), (5, 1), (1, 5)\}$ **no**
7. $\{(3, -1), (-2, 3), (-1, -5), (3, 2)\}$ **no**
8. $\{(-2, 9), (3, 9), (-0.5, 9), (4, 9)\}$ **yes**

Example 3 (page 259)

Make a table for each function. Use 1, 2, 3, and 4 for the domain. **9-16. See back of book.**

9. $f(x) = x + 7$
10. $y = 11x - 1$
11. $f(x) = x^2$
12. $f(x) = -4x$
13. $f(x) = 15 - x$
14. $y = 3x + 2$
15. $y = \frac{1}{4}x$
16. $f(x) = -x + 2$

Example 4 (page 259)

Find the range of the function rule $y = 5x - 2$ for each domain.

17. $\{0.5, 11\}$ **$\{0.5, 53\}$**
18. $\{-1.2, 0, 4\}$ **$\{-8, -2, 18\}$**
19. $\{-5, -1, 0, 2, 10\}$ **$\{-27, -7, -2, 8, 48\}$**
20. $\left\{-\frac{1}{2}, \frac{1}{4}, \frac{2}{5}\right\}$ **$\left\{-4\frac{1}{2}, -\frac{3}{4}, 0\right\}$**

PowerPoint

Additional Examples

1 Determine whether each relation is a function.

a. $\{(4, 3), (2, -1), (-3, -3), (2, 4)\}$ **not a function**

b. $\{(-4, 0), (2, 12), (-1, -3), (1, 5)\}$ **function**

2 Use the vertical-line test to determine whether the relation $\{(3, 2), (5, -1), (-5, 3), (-2, 2)\}$ is a function. **function**

3 Make a table for $f(t) = 0.5t + 1$. Use 1, 2, 3, and 4 as domain values.

t	$0.5t + 1$	$f(t)$
1	0.5(1) + 1	1.5
2	0.5(2) + 1	2
3	0.5(3) + 1	2,5
4	0.5(4) + 1	3

4 EXAMPLE Alternative Method

Another way to find the values of the range is to input the function rule into the Y= function of the calculator. In TblSet, start at -3 and count by 1. Then press 2nd TABLE. Use the arrow keys to scroll up and down the table to find the necessary domain/range pairs.

PowerPoint

Additional Examples

4 Evaluate the function rule $f(g) = -2g + 4$ to find the range for the domain $\{-1, 3, 5\}$. **$\{-6, -2, 6\}$**

Resources

- Daily Notetaking Guide 5-2 L3
- Daily Notetaking Guide 5-2—Adapted Instruction L1

Closure

Have students explain the difference between a relation and a function. **A relation is any set of ordered pairs. A function is a set of ordered pairs in which no x-value repeats with a different y-value.**

3. Practice

Assignment Guide

1 A B	1-8, 21-26, 32-35, 37-40	
2 A B	9-20, 27-31, 36, 41-42	
C Challenge		43-48
Test Prep		49-52
Mixed Review		53-63

Homework Quick Check

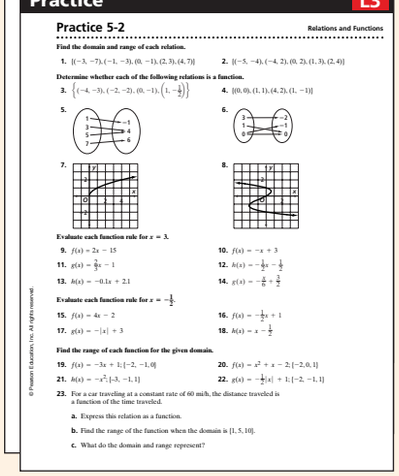
To check students' understanding of key skills and concepts, go over Exercises 21, 24, 31, 41, 42.

Error Prevention!

Exercise 18 Remind students to look at the values carefully. Since the values are written in a list with commas between them, they may see −1.2 as −1, 2 if they are not careful.

Differentiated Instruction Resources

- GPS Guided Problem Solving L3
- Enrichment L4
- Reteaching L2
- Adapted Practice L1
- Practice L3


Practice 5-2 Relations and Functions

Find the domain and range of each relation.

1. $\{(-3, -7), (-1, -3), (0, -1), (2, 3), (4, 7)\}$
2. $\{(-5, -4), (-4, 2), (0, 2), (1, 3), (2, 4)\}$

Determine whether each of the following relations is a function.

3. $\left\{(-4, -3), (-2, -2), (0, -1), \left(1, -\frac{1}{2}\right)\right\}$
4. $\{(0, 0), (1, 1), (4, 2), (1, -1)\}$
5.
6.
7.
8.

Evaluate each function rule for $x = 3$.

9. $f(x) = 2x - 15$
10. $f(x) = -x + 3$
11. $g(x) = \frac{2}{3}x - 1$
12. $h(x) = -\frac{1}{3}x - \frac{1}{2}$
13. $h(x) = -0.1x + 2.1$
14. $g(x) = -\frac{2}{5} + \frac{3}{5}$

Evaluate each function rule for $x = -\frac{1}{2}$.

15. $f(x) = 4x - 2$
16. $f(x) = -\frac{1}{2}x + 1$
17. $g(x) = -|x| + 3$
18. $h(x) = x - \frac{1}{2}$

Find the range of each function for the given domain.

19. $f(x) = -3x + 1; \{-2, -1, 0\}$
20. $f(x) = x^2 + x - 2; \{-2, 0, 1\}$
21. $h(x) = -x^2; \{-3, -1, 1\}$
22. $g(x) = -\frac{1}{2}|x| + 1; \{-2, -1, 1\}$
23. For a car traveling at a constant rate of 60 mi/h, the distance traveled is a function of the time traveled.
    a. Express this relation as a function.
    b. Find the range of the function when the domain is $\{1, 5, 10\}$.
    c. What do the domain and range represent?

© Pearson Education, Inc. All rights reserved.

B Apply Your Skills

Determine whether each relation is a function. If the relation is a function, state the domain and range.

21.

x	y
1	−3
6	−2
9	−1
1	3

no

22.

x	y
0	2
3	1
3	−1
5	3

no

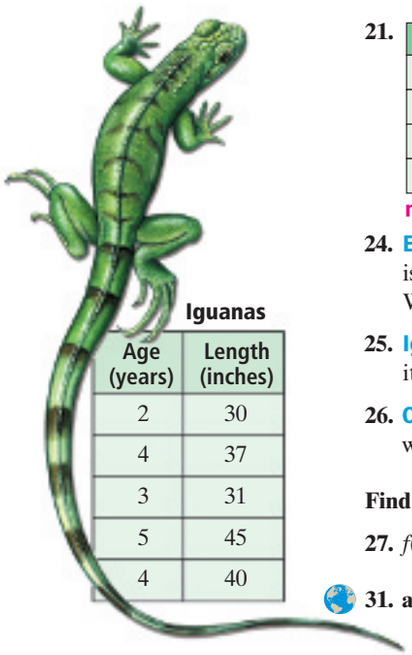
23.

x	y
−4	−4
−1	−4
0	−4
3	−4

yes; {−4, −1, 0, 3}; {−4}

24. Error Analysis A student thinks that the relation $\{(2, 1), (3, -2), (4, 5), (5, -2)\}$ is not a function because two values in the domain have the same range value. What is the student's error? See margin.

25. Iguanas Use the data in the table at the left. Is an iguana's length a function of its age? Explain. No; two 4-year-old iguanas may have different lengths.



Iguanas

Age (years)	Length (inches)
2	30
4	37
3	31
5	45
4	40

26. Open-Ended Create a data table for a relation that is *not* a function. Describe what your data might represent. See margin p. 261.

Find the range of each function for the domain {−1, 0.5, 3.7}. 27–30. See below left.

27. $f(x) = 4x + 1$ **28.** $g(x) = -4x + 1$ **29.** $y = |x| - 1$ **30.** $s(t) = t^2 - 1$

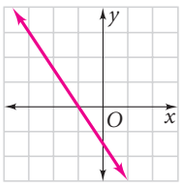
27. {−3, 3, 15.8}
28. {−13.8, −1, 5}
29. {−0.5, 0, 2.7}
30. {−0.75, 0, 12.69}

31. a. Profit A store bought a case of disposable cameras for $300. The store's profit p on the cameras is a function of the number c of cameras sold. Find the range of the function $p = 6c - 300$ when the domain is $\{0, 15, 50, 62\}$.

b. Critical Thinking In this situation, what do the domain and range represent? a–b. See margin.

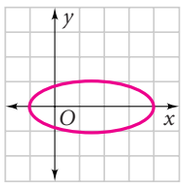
Determine whether each graph is the graph of a function.

32.



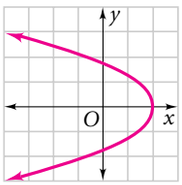
yes

33.



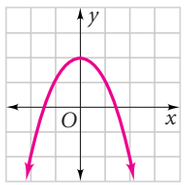
no

34.



no

35.



yes

36. Physics Light travels about 186,000 miles per second. The rule $d = 186{,}000t$ describes the relationship between distance d in miles and time t in seconds.

a. How far does light travel in 20 seconds? 3,720,000 mi

b. How far does light travel in 1 minute? 11,160,000 mi

For Exercises 37–40 assume that each variable has a different value. Determine whether each relation is a function.

37. $\{(a, b), (b, a), (c, c), (e, d)\}$ yes

38. $\{(b, b), (c, d), (d, c), (c, a)\}$ no

39. $\{(c, e), (c, d), (c, b)\}$ no

40. $\{(a, b), (b, c), (c, d), (d, e)\}$ yes

GO Online Homework Help
Visit: PHSchool.com
Web Code: ate-0502

24. Answers may vary. Sample: A relation is not a function if two range values have the same domain value.

26. Answers may vary. Sample:

x	y
14	60
13	58
16	60
14	63

Data represent the ages (x) and heights (y) of 4 students.

31a. {−300, −210, 0, 72}

b. Domain is the number of cameras sold, and range is the profit.

Real-World Connection

A telecommunications device for the deaf (TDD) includes a keyboard and a visual display of the conversation. This lets a hearing-impaired person use a telephone.

41. Telephone Bill The cost of a long-distance telephone call c is a function of the time spent talking t in minutes. The rule $c(t) = 0.09t$ describes the function for one service provider. At the right, a student has calculated how much a 2-hour phone call would cost.

$c = 0.09 \times 2$
$= 0.18$
\$.18 for 2 hours

a. Writing Why does the student's answer seem unreasonable? **a–b. See margin.**

b. Error Analysis What mistake(s) did the student make?

c. How much would it cost to make a 2-hour phone call? **\$10.80**

d. Critical Thinking What set of numbers is reasonable for the domain values? For the range values? **whole numbers; positive numbers**

42. Travel Suppose your family is driving home from vacation. The car averages 25 miles per gallon, and you are 180 miles from home. The function $d = 180 - 25g$ relates the number of gallons of gas g the car will use to your distance from home d. **a-c. See back of book.**

a. Make a table for $d = 180 - 25g$. Use 2, 4, 6, and 8 as domain values.

b. Estimation Based on the table, how many gallons of gasoline are needed to get home?

c. The gas tank holds 15 gallons when it is full. Describe a reasonable domain and range for this situation. Explain your answer.

Challenge

Use the functions $f(x) = 2x$ and $g(x) = x^2 + 1$ to find the value of each expression.

43. $f(3) + g(4)$ **23** **44.** $g(3) + f(4)$ **18** **45.** $f(5) - 2g(1)$ **6** **46.** $f(g(3))$ **20**

47. Critical Thinking Can the graph of a function be a horizontal line? A vertical line? Explain why or why not. **See left.**

47. Yes, it passes the vertical-line test; no, it doesn't pass the vertical-line test.

48. The function $y = [x]$ is called the *greatest-integer function*. $[x]$ is the greatest integer less than or equal to x. For example, $[2.99] = 2$ and $[-2.3] = -3$.

a. Evaluate the function for 0.5, −0.1, −1.99, and −5.2. **0, −1, −2, −6**

b. The domain of $y = [x]$ is all real numbers. What is the range of $y = [x]$? **all integers**

Test Prep

Gridded Response

49. Evaluate the function rule $f(x) = 7x$ for $x = 0.75$. **5.25**

50. Evaluate the function rule $f(x) = 9 - 0.2x$ for $x = 1.5$. **8.7**

51. What is the greatest value in the range of $y = x^2 - 7$ for the domain $\{-2, 0, 1\}$? **−3**

Short Response

52. Determine whether the data below are a function. Show your work. **See margin.**

Mount Rushmore Temperatures (°F)

At Base of Mountain	At Top of Mountain
80	72
65	58
93	84
98	91
74	69

41a. Answers may vary. Sample: The cost appears to be far too little.

b. Answers may vary. Sample: The student failed to convert hours to minutes.

52. [2] Domain → Range: 65 → 58, 74 → 69, 80 → 72, 93 → 84, 98 → 91 **(OR graph shown) Yes, the data represent a function.**

[1] shows calculation but no mapping diagram or graph

4. Assess & Reteach

PowerPoint

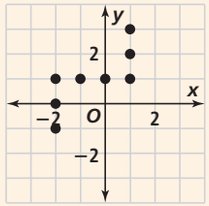
Lesson Quiz

1. a. Find the domain and range of the ordered pairs (1, 3), (−4, 0), (3, 1), (0, 4), (2, 3). **domain: {−4, 0, 1, 2, 3} range: {0, 1, 3, 4}**

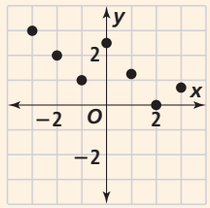
b. Use mapping to determine whether the relation is a function. **The relation is a function.**

2. Use the vertical-line test to determine whether each relation is a function. **a. no; b. yes**

a.



b.



3. Find the range of the function $f(g) = 3g - 5$ for the domain $\{-1.5, 2, 4\}$. **{−9.5, 1, 7}**

Alternative Assessment

Organize students into groups of three. Have students draw a function machine such as the one shown in Lesson 5-2. One student writes a function rule on a sticky note and places it on the machine. Another student chooses a domain value, writes it on a sticky note and places it at the input. The third student finds the corresponding range value. Continue until each student has used a different domain value to find a range value. Repeat the whole process two more times, each time allowing a different student to write the function rule.

Test Prep

A sheet of blank grids is available in the Test-Taking Strategies with Transparencies booklet. Give this sheet to students for practice with filling in the grids.

Resources
For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 303
- Test-Taking Strategies, p. 298
- Test-Taking Strategies with Transparencies

Exercise 51 Point out to students that since any number squared is positive they should choose the domain value with the greatest absolute value.

Checkpoint Quiz

Use this Checkpoint Quiz to check students' understanding of the skills and concepts of Lessons 5-1 through 5-2.

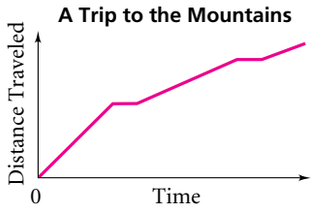
Resources
Grab & Go
- Checkpoint Quiz 1

Mixed Review

GO for Help

Lesson 5-1

53. The graph shows distance from home as a family drives to the mountains for a vacation. Copy the graph. Label each section of the graph. **See back of book.**



Lesson 3-5

The scale of a map is 1 in. : 15 mi. Find the actual distance corresponding to each map distance.

54. 2 in. **30 mi**
55. 1.5 in. **22.5 mi**
56. 0.5 in. **7.5 mi**
57. 3.25 in. **48.75 mi**
58. 5.5 in. **82.5 mi**
59. 7.25 in. **108.75 mi**

Lesson 1-7

Find the mean, median, mode, and range. **60–63. See left.**

60. 34 33 35 33 32 35 34 32 — **60. 33.5, 33.5, none, 3**

61. 1 −2 0 −1 1 −2 2 0 1 −2 — **61. $-\frac{1}{5}$, 0, −2 and 1, 4**

62. 4 5 3 7 1 12 6 9 5 — **62. $\frac{52}{9}$, 5, 5, 11**

63. 15 13 19 20 9 13 15 13 — **63. $\frac{117}{8}$, 14, 13, 11**

Checkpoint Quiz 1 — Lessons 5-1 through 5-2

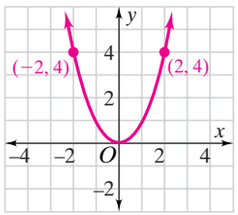
Sketch a graph of each situation. Label each section. **1–3. See margin.**

1. the height of a plant that grows at a steady rate

2. the temperature in a classroom after the heater is turned on

3. a child's height above the ground while on a swing

4. Is the graph at the right the graph of a function? Explain.
Yes; it passes the vertical-line test.



Make a table for each function. Use 1, 2, 3, and 4 for the domain. **5–8. See back of book.**

5. $f(x) = -5x$
6. $g(x) = x + 1.4$
7. $f(n) = 3n^2$
8. $y = 2 - 0.5x$

Determine whether each relation is a function.

9.

x	y
0	6
1	7
5	8
8	9

function

10.

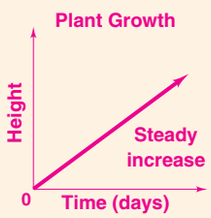
x	y
5	1
−6	8
5	3
6	7

not a function

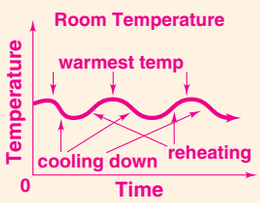
page 262 Checkpoint Quiz 1

1–3. Graphs may vary. Samples are given.

1.



2.



3.