A relation is a set of ordered pairs. The \((\text{age, height})\) ordered pairs below form a relation.

You can list the set of ordered pairs in a relation using braces.

\[
\{(18, 4.25), (20, 4.40), (21, 5.25), (14, 5.00), (18, 4.85)\}
\]

Recall from Lesson 1-4 that a function is a relation that assigns exactly one output (range) value for each input (domain) value.

One way you can tell if a relation is a function is by making a mapping diagram. List the domain values and the range values in order. Draw arrows from the domain values to their range values.

**EXAMPLE**

**Using a Mapping Diagram**

Determine whether each relation is a function.

a. \(\{(11, -2), (12, -1), (13, -2), (20, 7)\}\)

b. \(\{(-2, -1), (-1, 0), (6, 3), (-2, 1)\}\)

The relation is a function.

The relation is not a function.
Another way you can tell whether a relation is a function is to analyze the graph of the relation using the **vertical-line test**. If any vertical line passes through more than one point of the graph then for some value of \( x \) there is more than one value of \( y \). Therefore, the relation is not a function.

**Using the Vertical-Line Test**

Determine whether the relation \( \{(3, 0), (-2, 1), (0, -1), (-3, 2), (3, 2)\} \) is a function.

**Step 1** Graph the ordered pairs on a coordinate plane.

**Step 2** Pass a pencil across the graph as shown.

A vertical line would pass through \( (3, 0) \) and \( (3, 2) \). The relation is not a function.

Use the vertical-line test to determine whether each relation is a function.

**a.** \( \{(4, -2), (1, 2), (0, 1), (-2, 2)\} \)

**b.** \( \{(0, 2), (1, -1), (-1, 4), (0, -3), (2, 1)\} \)

**Evaluating Functions**

Recall from Lesson 1-4 that a function rule is an equation that describes a function. You can think of a function rule as an input-output machine.

The **domain** is the set of input values.

The **range** is the set of output values.

If you know the input values, you can use a function rule to find the output values. The output values depend on the input values.

Another way to write the function \( y = 3x + 4 \) is \( f(x) = 3x + 4 \). A function is in **function notation** when you use \( f(x) \) to indicate the outputs. You read \( f(x) \) as “\( f \) of \( x \)” or “\( f \) is a function of \( x \).” The notations \( g(x) \) and \( h(x) \) also indicate functions of \( x \).

In Lesson 1-4 you wrote function rules from tables. You can also make a table using values from a function rule.
**Vocabulary Tip**

You can think of the notation \( f(6) \) as “Replace \( n \) with 6 to find the value of \( f(6) \).”

---

**EXERCISES**

**Practice and Problem Solving**

**A** Practice by Example

**Example 1** (page 257)

1. \( \{(3, 7), (3, 8), (3, -2), (3, 4), (3, 1)\} \) no
2. \( \{(-6, -7), (5, -8), (1, 4), (5, 5)\} \) no
3. \( \{(0.04, 0.2), (0.2, 1), (1, 5), (5, 25)\} \) yes
4. \( \{(4.2, 2.1), (0.0, 0.1), (1, -1), (4, -2)\} \) no

**Example 2** (page 258)

Use the vertical-line test to determine whether each relation is a function.

5. \( \{(2.5, 3.5), (4.5, 5.5)\} \) yes
6. \( \{(5.0, 0.5), (5.1, 1.5)\} \) no
7. \( \{(3, -1), (-2, 3), (-1, -5), (3, 2)\} \) no
8. \( \{(-2.9, 3.9), (-0.5, 9), (4.9)\} \) yes

**Example 3** (page 259)

Make a table for each function. Use 1, 2, 3, and 4 for the domain.

9. \( f(x) = x + 7 \)
10. \( y = 11x - 1 \)
11. \( f(x) = x^2 \)
12. \( f(x) = -4x \)
13. \( f(x) = 15 - x \)
14. \( y = 3x + 2 \)
15. \( y = \frac{1}{4}x \)
16. \( f(x) = -x + 2 \)

**Example 4** (page 259)

Find the range of the function rule \( y = 5x - 2 \) for each domain.

17. \( \{0.5, 11\} \)
18. \( \{-12, 0, 4\} \)
19. \( \{-5, -1, 0, 2, 10\} \)
20. \( \{-\frac{1}{2}, \frac{1}{2}, \frac{7}{2}\} \)

---

**Quick Check**

3. Make a table for \( y = 8 - 3x \). Use 1, 2, 3, and 4 as domain values. See back of book.

You can use a function rule and a given domain to find the range of the function. After computing the range values, write the values in order from least to greatest.

**Finding the Range**

Evaluate the function rule \( f(a) = -3a + 5 \) to find the range of the function for the domain \([-3, 1, 4]\).

\[
\begin{align*}
f(a) & = -3a + 5 \\
f(-3) & = -3(-3) + 5 \\
f(-3) & = 14 \\
\text{The range is } & \{-7, 2, 14\}.
\end{align*}
\]

**Quick Check**

4. Find the range of each function for the domain \([-2, 0, 5]\).
   a. \( f(x) = x - 6 \)
   b. \( y = -4x \)
   c. \( g(t) = t^2 + 1 \)

\[\{(-8, -6, -1)\}, \{(-20, 0, 8)\}, \{(1, 5, 26)\}\]

**EXTRA SKILL AND WORD PROBLEM PRACTICE**

For more exercises, see Extra Skill and Word Problem Practice.
Determine whether each relation is a function. If the relation is a function, state the domain and range.

21. \[
\begin{array}{c|c}
 x & y \\
 1 & -3 \\
 6 & -2 \\
 9 & -1 \\
 1 & 3 \\
\end{array}
\]

22. \[
\begin{array}{c|c}
 x & y \\
 0 & 2 \\
 3 & 1 \\
 3 & -1 \\
 5 & 3 \\
\end{array}
\]

23. \[
\begin{array}{c|c}
 x & y \\
 -4 & -4 \\
 -1 & -4 \\
 0 & -4 \\
 3 & -4 \\
\end{array}
\]

24. Error Analysis A student thinks that the relation \{(2, 1), (3, -2), (4, 5), (5, -2)\} is not a function because two values in the domain have the same range value. What is the student’s error? See margin.

25. Iguanas Use the data in the table at the left. Is an iguana’s length a function of its age? Explain. No; two 4-year-old iguanas may have different lengths.


Find the range of each function for the domain \([-1, 0.5, 3, 7]\).

27. \(f(x) = 4x + 1\)  
28. \(g(x) = -4x + 1\)  
29. \(y = |x| - 1\)  
30. \(s(t) = t^2 - 1\)

31. a. Profit A store bought a case of disposable cameras for $300. The store’s profit \(p\) on the cameras is a function of the number \(c\) of cameras sold. Find the range of the function \(p = 6c - 300\) when the domain is \([0, 15, 50, 62]\).

b. Critical Thinking In this situation, what do the domain and range represent? A-c. See margin.

Determine whether each graph is the graph of a function.

32. yes  
33. no

34. no  
35. yes

36. Physics Light travels about 186,000 miles per second. The rule \(d = 186,000t\) describes the relationship between distance \(d\) in miles and time \(t\) in seconds.

a. How far does light travel in 20 seconds? 3,720,000 mi

b. How far does light travel in 1 minute? 11,160,000 mi

For Exercises 37–40 assume that each variable has a different value. Determine whether each relation is a function.

37. \([(a, b), (b, a), (c, c), (e, d)]\) yes  
38. \([(b, b), (c, d), (d, c), (c, a)]\) no  
39. \([(c, e), (c, d), (c, b)]\) no  
40. \([(a, b), (b, c), (c, d), (d, e)]\) yes

24. Answers may vary. Sample: A relation is not a function if two range values have the same domain value.

26. Answers may vary. Sample: 

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>60</td>
</tr>
<tr>
<td>13</td>
<td>58</td>
</tr>
<tr>
<td>16</td>
<td>60</td>
</tr>
<tr>
<td>14</td>
<td>63</td>
</tr>
</tbody>
</table>

Data represent the ages (\(x\)) and heights (\(y\)) of 4 students.

31a. \([-300, -210, 0, 72]\)

b. Domain is the number of cameras sold, and range is the profit.
41. Telephone Bill  The cost of a long-distance telephone call $c$ is a function of the time spent talking $t$ in minutes. The rule $c(t) = 0.09t$ describes the function for one service provider. At the right, a student has calculated how much a 2-hour phone call would cost.

   a. Writing  Why does the student's answer seem unreasonable?  a-b. See margin.
   b. Error Analysis  What mistake(s) did the student make?
   c. How much would it cost to make a 2-hour phone call?  $10.80$
   d. Critical Thinking  What set of numbers is reasonable for the domain values? For the range values?

   42. Travel  Suppose your family is driving home from vacation. The car averages 25 miles per gallon, and you are 180 miles from home. The function $d = 180 - 25g$ relates the number of gallons of gas $g$ the car will use to travel your distance from home $d$.  a-c. See back of book.
   a. Make a table for $d = 180 - 25g$. Use 2, 4, 6, and 8 as domain values.
   b. Estimation  Based on the table, how many gallons of gasoline are needed to get home?
   c. The gas tank holds 15 gallons when it is full. Describe a reasonable domain and range for this situation. Explain your answer.

   Use the functions $f(x) = 2x$ and $g(x) = x^2 + 1$ to find the value of each expression.

   43. $f(3) + g(4)$  23  44. $g(3) + f(4)$  18  45. $f(5) - 2g(1)$  6  46. $f(g(3))$  20

   47. Critical Thinking  Can the graph of a function be a horizontal line? A vertical line? Explain why or why not.  See left.

   48. The function $y = [x]$ is called the greatest-integer function. $[x]$ is the greatest integer less than or equal to $x$. For example, $[2.99] = 2$ and $[-2.3] = -3$.
   a. Evaluate the function for $0.5, 0.1, 1.99$, and $-5.2$. 0, 1, 2, 3
   b. The domain of $y = [x]$ is all real numbers. What is the range of $y = [x]$? all integers

   49. Evaluate the function rule $f(x) = 7x$ for $x = 0.75$.  5.25
   50. Evaluate the function rule $f(x) = 9 - 0.2x$ for $x = 1.5$.  8.7
   51. What is the greatest value in the range of $y = x^2 - 7$ for the domain $(-2, 0, 1)$?  0

   52. Determine whether the data below are a function. Show your work.

   Mount Rushmore Temperatures (°F)

<table>
<thead>
<tr>
<th>At Base of Mountain</th>
<th>At Top of Mountain</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>72</td>
</tr>
<tr>
<td>65</td>
<td>58</td>
</tr>
<tr>
<td>93</td>
<td>84</td>
</tr>
<tr>
<td>98</td>
<td>91</td>
</tr>
</tbody>
</table>

   41a. Answers may vary. Sample: The cost appears to be far too little.

   b. Answers may vary. Sample: The student failed to convert hours to minutes.

   52. Domain  Range
   | 95 | 58 |
   | 74 | 89 |
   | 80 | 72 |
   | 93 | 84 |
   | 98 | 91 |

   (OR graph shown) Yes, the data represent a function. [1] shows calculation but no mapping diagram or graph

   Alternative Assessment

   Organize students into groups of three. Have students draw a function machine such as the one shown in Lesson 5-2. One student writes a function rule on a sticky note and places it on the machine. Another student chooses a domain value, writes it on a sticky note and places it at the input. The third student finds the corresponding range value. Continue until each student has used a different domain value to find a range value. Repeat the whole process two more times, each time allowing a different student to write the function rule.
Mixed Review

Lesson 5-1

53. The graph shows distance from home as a family drives to the mountains for a vacation. Copy the graph. Label each section of the graph.  See back of book.

Lesson 3-5

The scale of a map is 1 in. : 15 mi. Find the actual distance corresponding to each map distance.

54. 2 in. 30 mi
55. 1.5 in. 22.5 mi
56. 0.5 in. 7.5 mi
57. 3.25 in. 48.75 mi
58. 5.5 in. 82.5 mi
59. 7.25 in. 108.75 mi

Lesson 1-7

Find the mean, median, mode, and range. 60–63. See left.

60. 33.5, 33.5, none, 3
61. –1, 0, –2 and 1, 4
62. \( \frac{52}{9}, 5, 5, 11 \)
63. \( \frac{117}{9}, 14, 13, 11 \)

Checkpoint Quiz 1

Sketch a graph of each situation. Label each section. 1–3. See margin.

1. the height of a plant that grows at a steady rate
2. the temperature in a classroom after the heater is turned on
3. a child’s height above the ground while on a swing

4. Is the graph at the right the graph of a function? Explain. Yes; it passes the vertical-line test.

Make a table for each function. Use 1, 2, 3, and 4 for the domain. 5–8. See back of book.

5. \( f(x) = -5x \)
6. \( g(x) = x + 1.4 \)
7. \( f(n) = 3n^2 \)
8. \( y = 2 - 0.5x \)

Determine whether each relation is a function.

9. function

10. not a function

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>–6</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>