6-5

1. Plan

Objectives
1 To graph and write linear equations using point-slope form
2 To write a linear equation using data

Examples
1 Graphing Using Point-Slope Form
2 Writing an Equation in Point-Slope Form
3 Using Two Points to Write an Equation
4 Writing an Equation Using a Table
5 Real-World Problem Solving

Professional Development

Math Background

No matter what form a linear equation is written in, the coordinates of every point on the graph of the line satisfy its linear equation and, conversely, every pair of values that satisfy the equation represent a point on the graph.

More Math Background: p. 306D

Lesson Planning and Resources

See p. 306E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:

Rate of Change and Slope
Lesson 6-1: Example 1
Extra Skills and Word Problem Practice, Ch. 6

The Distributive Property
Lesson 2-4: Example 3
Extra Skills and Word Problem Practice, Ch. 2

6-5 Point-Slope Form and Writing Linear Equations

What You'll Learn
- To graph and write linear equations using point-slope form
- To write a linear equation using data

. . . And Why
To write an equation relating altitude and the boiling point of water, as in Example 5

Check Skills You'll Need

GO for Help Lessons 6-1 and 2-4

Find the rate of change of the data in each table.

1.

x	y
2	4
5	−2
8	−8
11	−14

−2

2.

x	y
−3	−5
−1	−4
1	−3
3	−2

$\frac{1}{2}$

3.

x	y
10	4
7.5	−1
5	−6
2.5	−11

2

Simplify each expression.

4. $-3(x - 5)$ **$-3x + 15$**
5. $5(x + 2)$ **$5x + 10$**
6. $-\frac{4}{9}(x - 6)$ **$-\frac{4}{9}x + \frac{8}{3}$**

New Vocabulary • point-slope form

1 Using Point-Slope Form

You can use the definition of slope to find another form of a linear equation called the point-slope form.

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$ **Use the definition of slope.**

Suppose you know that a line passes through the point (3, 4) and has slope 2.

$$\frac{y - 4}{x - 3} = 2$$ **Substitute (3, 4) for (x_1, y_1) and substitute (x, y) for (x_2, y_2). Substitute 2 for m.**

$$\frac{y - 4}{x - 3}(x - 3) = 2(x - 3)$$ **Multiply each side by $x - 3$.**

$$y - 4 = 2(x - 3)$$ **Simplify the left side of the equation.**

The equation $y - 4 = 2(x - 3)$ is in point-slope form.

$$y - 4 = 2(x - 3)$$

y-coordinate slope *x*-coordinate

Key Concepts

Definition **Point-Slope Form of a Linear Equation**

The **point-slope form** of the equation of a nonvertical line that passes through the point (x_1, y_1) and has slope m is

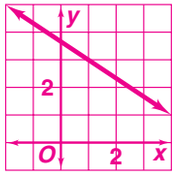
$$y - y_1 = m(x - x_1)$$

Differentiated Instruction Solutions for All Learners

Special Needs L1
In Example 2, to help any confusion about the signs, students graph a line and label two points. Then they write an equation for slope using the coordinates of the points. Ask: *What operation do you use to find the rise or the run?* **subtraction**
learning style: verbal

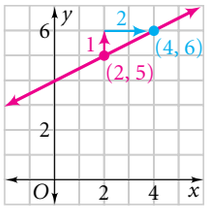
Below Level L2
In Step 2 of Example 4, have students write equations in point-slope form using each ordered pair in the table. Then ask them to show that all four equations are equivalent.
learning style: verbal

1.



1 EXAMPLE Graphing Using Point-Slope Form

Graph the equation $y - 5 = \frac{1}{2}(x - 2)$.



The equation shows that the line passes through (2, 5) and has a slope $\frac{1}{2}$.

Start at (2, 5). Using the slope, go up 1 unit and right 2 units to (4, 6). Draw a line through the two points.

Quick Check 1 Graph the equation $y - 5 = -\frac{2}{3}(x + 2)$. **See left.**

2 EXAMPLE Writing an Equation in Point-Slope Form

Vocabulary Tip

Square brackets, [ ], are grouping symbols commonly used when parentheses, ( ), are inside.

Write the equation of the line that has slope -3 that passes through the point $(-1, 7)$.

$y - y_1 = m(x - x_1)$ **Use the point-slope form.**

$y - 7 = -3[x - (-1)]$ **Substitute $(-1, 7)$ for (x_1, y_1) and -3 for m.**

$y - 7 = -3(x + 1)$ **Simplify inside the grouping symbols.**

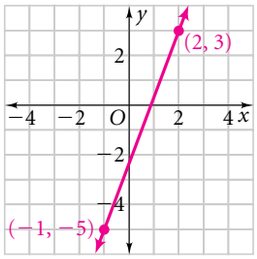
Quick Check 2 Write the equation of the line that has slope $\frac{2}{5}$ that passes through the point $(10, -8)$.
$y + 8 = \frac{2}{5}(x - 10)$

If you know two points on a line, first use them to find the slope. Then you can write an equation using either point.

3 EXAMPLE Using Two Points to Write an Equation

For: Point-Slope Activity
Use: Interactive Textbook, 6-5

Write equations for the line in point-slope form and in slope-intercept form.



Step 1 Find the slope.

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$

$$\frac{-5 - 3}{-1 - 2} = \frac{8}{3}$$

The slope is $\frac{8}{3}$.

Step 2 Use either point to write the equation in point-slope form. Use $(2, 3)$.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{8}{3}(x - 2)$$

Step 3 Rewrite the equation from Step 2 in slope-intercept form.

$$y - 3 = \frac{8}{3}(x - 2)$$

$$y - 3 = \frac{8}{3}x - 5\frac{1}{3}$$

$$y = \frac{8}{3}x - 2\frac{1}{3}$$

Quick Check 3 **a.** Write an equation for the line in Example 3 in point-slope form using the point $(-1, -5)$. $y + 5 = \frac{8}{3}(x + 1)$

b. Write the equation you found in part (a) in slope-intercept form. $y = \frac{8}{3}x - 2\frac{1}{3}$

c. What is true about the equation you wrote in part (b) and the equation in Step 3 of Example 3? **They are the same.**

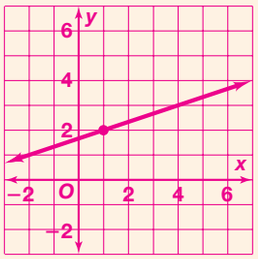
2. Teach

Guided Instruction

PowerPoint

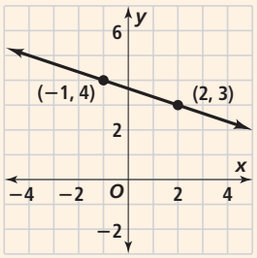
Additional Examples

1 Graph the equation $y - 2 = \frac{1}{3}(x - 1)$.



2 Write the equation of the line with slope -2 that passes through the point $(3, -3)$.
$y + 3 = -2(x - 3)$

3 Write equations for the line in point-slope form and in slope-intercept form.
$y - 4 = -\frac{1}{3}(x + 1)$
$y = -\frac{1}{3}x + 3\frac{2}{3}$



Advanced Learners L4
Have students write the slope-intercept form of the linear equation found in Quick Check of Example 5. Ask them what the *y*-intercept in the equation represents.

learning style: verbal

English Language Learners ELL
Be sure that students understand that when a linear equation is written in *Point-Slope* form, they can identify a *point* on, and the *slope* of the graph of the equation just by looking at the equation.

learning style: verbal

5 EXAMPLE **Connection to Science**

Boiling point is defined as the temperature at which the vapor pressure of a liquid slightly exceeds the pressure of the atmosphere above the liquid. Water at 1 atmosphere pressure boils at 212° F. If the pressure on a liquid is reduced, the boiling point is lowered. Air pressure is less at higher elevations. Denver, Colorado is 1 mile above sea level, so the boiling point of water there averages 201° F.

PowerPoint

Additional Examples

4 Is the relationship shown by the data linear? If so, model the data with an equation. **yes; $y - 4 = 2(x - 2)$**

x	y
3	6
2	4
−1	−2
−3	−6

5 Is the relationship shown by the data linear? If so, model the data with an equation. **no**

x	y
−2	−2
−1	−1
1	0
2	1

Resources

- Daily Notetaking Guide 6-5 L3
- Daily Notetaking Guide 6-5—Adapted Instruction L1

Closure

Ask students to write a set of data that is linear, and then model the data with an equation. Have them graph the data and the equation.

2 Writing Linear Equations Using Data

You can write a linear equation to model data in tables. Two sets of data have a linear relationship if the rate of change between consecutive pairs of data is the same. For data that have a linear relationship, the rate of change is the slope.

4 EXAMPLE Writing an Equation Using a Table

Is the relationship shown by the data linear? If so, model the data with an equation.

Step 1 Find the rate of change for consecutive ordered pairs.

x	y
−1	4
3	6
5	7
11	10

4, 2, 6 — 2, 1, 3

$\frac{2}{4} = \frac{1}{2}$ $\quad \frac{1}{2} = \frac{1}{2}$ $\quad \frac{3}{6} = \frac{1}{2}$

Step 2 Use the slope and a point to write an equation.

$y - y_1 = m(x - x_1)$

Substitute (5, 7) for (x_1, y_1) and $\frac{1}{2}$ for m.

$y - 7 = \frac{1}{2}(x - 5)$

Quick Check **4** Is the relationship shown by the data at the right linear? If so, model the data with an equation.

Yes; answers may vary. Sample: $y - 5 = \frac{2}{5}(x - 19)$

x	y
−11	−7
−1	−3
4	−1
19	5

5 EXAMPLE Real-World Problem Solving

Is the relationship shown by the data linear? If so, model the data with an equation.

Boiling Point of Water

Altitude (1000 ft)	Temperature (°F)
8	197.6
4.5	203.9
3	206.6
2.5	207.5

−3.5, −1.5, −0.5 — 6.3, 2.7, 0.9

Step 1 Find the rates of change for consecutive ordered pairs.

$\frac{6.3}{-3.5} = -1.8 \qquad \frac{2.7}{-1.5} = -1.8 \qquad \frac{0.9}{-0.5} = -1.8$

The relationship is linear. The rate of change is −1.8 degrees Fahrenheit per 1000 ft of altitude.

Step 2 Use the slope and a point to write an equation.

$y - y_1 = m(x - x_1)$ **Use the point-slope form.**

$y - 206.6 = -1.8(x - 3)$ **Substitute (3, 206.6) for (x_1, y_1) and −1.8 for m.**

The equation $y - 206.6 = -1.8(x - 3)$ relates altitude in thousands of feet x to the boiling point temperature in degrees Fahrenheit.

Real-World Connection

At 5280 feet above sea level it takes 17 minutes to hard-boil an egg. This is more than 40% longer than it takes the same egg to cook at sea level.

pages 339–341 Exercises

10. $y + 4 = 6(x - 3)$
11. $y - 2 = -\frac{5}{3}(x - 4)$
12. $y - 2 = \frac{4}{5}(x)$
13. $y + 7 = -\frac{3}{2}(x + 2)$
14. $y = 1(x - 4)$
15. $y + 8 = -3(x - 5)$
16. $y - 2 = 0(x + 5)$ or $y = 2$
17. $y + 8 = -\frac{1}{5}(x - 1)$
18. $y - 1 = \frac{2}{3}(x + 6)$
19–30. **Answers may vary from the point indicated by the equation.**
19. $y = 1(x + 1)$; $y = x + 1$
20. $y - 5 = \frac{5}{3}(x - 3)$; $y = \frac{5}{3}x$

Quick Check **5** Is the relationship shown by the data in the table linear? If it is, model the data with an equation.
Yes; answers may vary. Sample:
$y - 3030 = -\frac{50}{3}(x - 68)$

Working Outdoors

Temperature	Calories Burned per Day
68°F	3030
62°F	3130
56°F	3230
50°F	3330

In Example 5 you could rewrite $y - 206.6 = -1.8(x - 3)$ as $y = -1.8x + 212$. This form gives you useful information about the y-intercept. For instance, 212°F is the boiling point of water at sea level.

Here are the three forms of linear equations you have studied.

Key Concepts

GO for Help
For more help with the three forms of a linear equation, see page 342.

Summary **Linear Equations**

Slope-Intercept Form	Standard Form	Point-Slope Form
$y = mx + b$	$Ax + By = C$	$(y - y_1) = m(x - x_1)$
m is the slope and b is the y-intercept.	A and B are not both 0.	(x_1, y_1) lies on the graph of the equation, and m is the slope.
Examples		
$y = -\frac{2}{3}x + \frac{5}{3}$	$2x + 3y = 5$	$y - 1 = -\frac{2}{3}(x - 1)$

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice.*

Practice and Problem Solving

A Practice by Example

GO for Help

Example 1 (page 337)

Graph each equation. 1–9. See back of book.

1. $y - 2 = (x - 3)$
2. $y - 2 = 2(x - 3)$
3. $y - 2 = -\frac{3}{2}(x - 3)$
4. $y + 5 = -(x - 2)$
5. $y + 1 = \frac{2}{3}(x + 4)$
6. $y - 1 = -3(x + 2)$
7. $y + 3 = -2(x - 1)$
8. $y - 4 = (x - 5)$
9. $y - 2 = 3(x + 2)$

Example 2 (page 337)

Write an equation in point-slope form for the line through the given point that has the given slope. 10–18. See margin.

10. $(3, -4); m = 6$
11. $(4, 2); m = -\frac{5}{3}$
12. $(0, 2); m = \frac{4}{5}$
13. $(-2, -7); m = -\frac{3}{2}$
14. $(4, 0); m = 1$
15. $(5, -8); m = -3$
16. $(-5, 2); m = 0$
17. $(1, -8); m = -\frac{1}{5}$
18. $(-6, 1); m = \frac{2}{3}$

Example 3 (page 337)

A line passes through the given points. Write an equation for the line in point-slope form. Then rewrite the equation in slope-intercept form. 19–30. See margin.

19. $(-1, 0), (1, 2)$
20. $(3, 5), (0, 0)$
21. $(4, -2), (9, -8)$
22. $(6, -4), (-3, 5)$
23. $(-1, -5), (-7, -6)$
24. $(-3, -4), (3, -2)$
25. $(2, 7), (1, -4)$
26. $(-2, 6), (5, 1)$
27. $(3, -8), (-2, 5)$
28. $\left(1, \frac{1}{2}\right), (3, 2)$
29. $\left(\frac{1}{2}, 2\right), \left(-\frac{3}{2}, 4\right)$
30. $(0.2, 1.1), (7, 3)$

21. $y + 2 = -\frac{6}{5}(x - 4)$; $y = -\frac{6}{5}x + \frac{14}{5}$
22. $y + 4 = -1(x - 6)$; $y = -x + 2$
23. $y + 5 = \frac{1}{6}(x + 1)$; $y = \frac{1}{6}x - \frac{29}{6}$
24. $y + 4 = \frac{1}{3}(x + 3)$; $y = \frac{1}{3}x - 3$
25. $y - 7 = 11(x - 2)$; $y = 11x - 15$
26. $y - 6 = -\frac{5}{7}(x + 2)$; $y = -\frac{5}{7}x + 4\frac{4}{7}$

3. Practice

Assignment Guide

1 A B	1-30, 36-53, 56-59
2 A B	31-35, 54, 55, 60
C Challenge	61-64
Test Prep	65-69
Mixed Review	70-81

Homework Quick Check

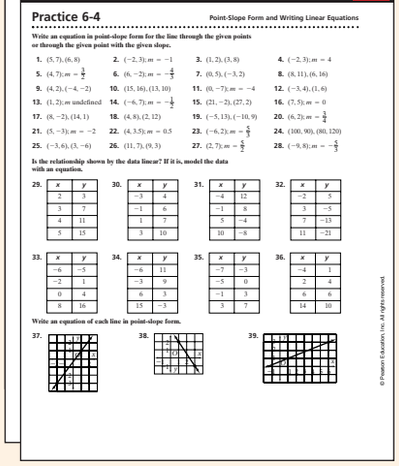
To check students' understanding of key skills and concepts, go over Exercises 18, 34, 40, 55, 57.

Error Prevention!

Exercises 19–30 Suggest to students that they circle the x-coordinates and box the y-coordinates.

Differentiated Instruction Resources

- GPS Guided Problem Solving L3
- Enrichment L4
- Reteaching L2
- Adapted Practice L1
- Practice L3


Practice 6-4 Point-Slope Form and Writing Linear Equations

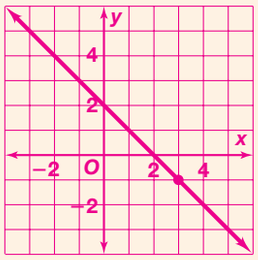
27. $y + 8 = -\frac{13}{5}(x - 3)$; $y = -\frac{13}{5}x - \frac{1}{5}$
28. $y - \frac{1}{2} = \frac{3}{4}(x - 1)$; $y = \frac{3}{4}x - \frac{1}{4}$
29. $y - 2 = -1(x - \frac{1}{2})$; $y = -x + \frac{5}{2}$
30. $y - 1.1 = \frac{1.9}{6.8}(x - 0.2)$; $y = \frac{1.9}{6.8}x + \frac{7.1}{6.8}$

4. Assess & Reteach

PowerPoint

Lesson Quiz

1. Graph the equation $y + 1 = -(x - 3)$.



2. Write an equation of the line with slope $-\frac{2}{3}$ that passes through the point (0, 4). $y - 4 = -\frac{2}{3}(x - 0)$, or $y = -\frac{2}{3}x + 4$

3. Write an equation for the line that passes through (3, −5) and (−2, 1) in point-slope form and slope-intercept form. $y + 5 = -\frac{6}{5}(x - 3)$; $y = -\frac{6}{5}x - \frac{7}{5}$

4. Is the relationship shown by the data linear? If so, model that data with an equation. yes; $y + 3 = \frac{2}{5}(x - 0)$

x	y
−10	−7
0	−3
5	−1
20	5

Alternative Assessment

Let students work in pairs. Instruct each student to write coordinates for two points, and then write an equation passing through those points in point-slope form. Have each pair exchange points and repeat. Then compare answers to check.

Example 4 (page 338)

31. Yes; answers may vary. Sample: $y - 9 = -2(x + 4)$

32. Yes; answers may vary. Sample: $y - 40 = 3(x - 5)$

Is the relationship shown by the data linear? If so, model the data with an equation.

31. See left.

x	y
−4	9
2	−3
5	−9
9	−17

32. See left.

x	y
−10	−5
−2	19
5	40
11	58

33. no

x	y
3	1
6	4
9	13
15	49

Example 5 (page 338)

34. Yes; answers may vary. Sample: $y - 75 = 10(x - 10)$

34. See left.

Speed Over Posted Speed (mi/h)	Fine ($)
10	75
12	95
15	125
19	165

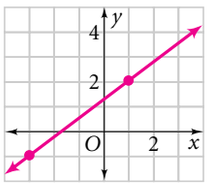
35. no

Volume (gal)	Weight (lb)
0	0
2	16
4	33
6	50

B Apply Your Skills

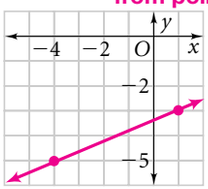
Write an equation of each line in point-slope form. 36–53. Answers may vary from point indicated by the equation.

36.



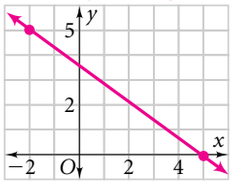
$y - 2 = \frac{3}{4}(x - 1)$

37.



$y + 3 = \frac{2}{5}(x - 1)$

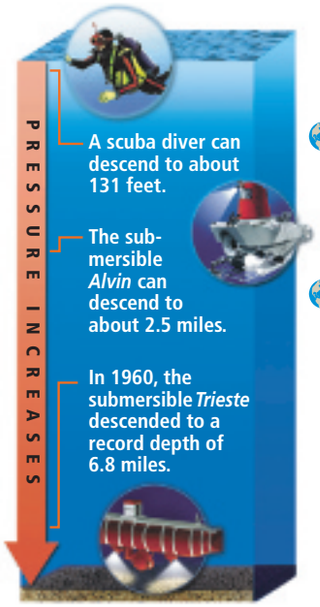
38.



$y = -\frac{5}{7}(x - 5)$

Write one equation of the line through the given points in point-slope form and one in standard form using integers. 39–53. See margin.

39. (1, 4), (−1, 1)
40. (6, −3), (−2, −3)
41. (0, 0), (−1, −2)
42. (0, 2), (−4, 2)
43. (−6, 6), (3, 3)
44. (2, 3), (−1, 5)
45. (5, −3), (3, 4)
46. (2, 2), (−1, 7)
47. (−7, 1), (5, −1)
48. (−8, 4), (−4, −2)
49. (2, 4), (−3, −6)
50. (5, 3), (4, 5)
51. (0, 1), (−3, 0)
52. (−2, 4), (0, −5)
53. (6, 2), (1, −1)



54. **Science** At the surface of the ocean, pressure is 1 atmosphere. At 66 ft below sea level, the pressure is 3 atmospheres. The relationship of pressure and depth is linear.
   a. Write an equation for the data. $y = -\frac{1}{33}x + 1$
   b. Predict the pressure at 100 ft below sea level. about 4 atmospheres

55. **Environment** Worldwide carbon monoxide emissions are decreasing about 2.6 million metric tons each year. In 1991, carbon monoxide emissions were 79 million metric tons. Use a linear equation to model the relationship between carbon monoxide emissions and time. Let $x = 91$ correspond to 1991. $y = -2.6x + 315.6$

56. a. **Open-Ended** Write an equation in point-slope form that contains the point (−4, −6). Explain your steps. a–b. See margin.
   b. How many equations could you write in part(a)? Explain.

57. **Critical Thinking** How would the graph of $y - 12 = 8(x - 2)$ change if all of the subtraction signs were changed to addition signs? y-intercept changes from −4 to 4.

58. **Reasoning** Is $y - 5 = 2(x - 1)$ an equation of a line through (4, 11)? Explain. Yes; the point satisfies the equation.

pages 307–309 Exercises

39. $y - 4 = \frac{3}{2}(x - 1)$; $-3x + 2y = 5$

40. $y + 3 = 0(x - 6)$; $y = -3$

41. $y + 2 = 2(x + 1)$; $-2x + y = 0$

42. $y - 2 = 0(x - 0)$; $y = 2$

43. $y - 6 = -\frac{1}{3}(x + 6)$; $x + 3y = 12$

44. $y - 3 = -\frac{2}{3}(x - 2)$; $2x + 3y = 13$

45. $y + 3 = -\frac{7}{2}(x - 5)$; $7x + 2y = 29$

46. $y - 2 = -\frac{5}{3}(x - 2)$; $5x + 3y = 16$

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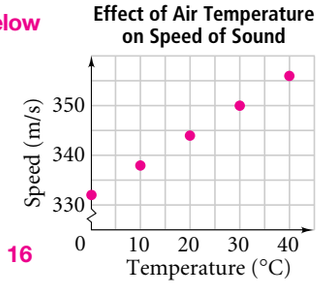
59. Open-Ended Write an equation in each of the following forms.
a. slope-intercept form $y = x + 1$
b. standard form $-x + y = 1$
c. point-slope form $y - 1 = 1(x - 0)$

59a-c. Answers may vary. Sample is given.

60. Science Use the scatter plot.
a. Write an equation to model the data. **See below left.**
b. What is the speed of sound at 15°C?
c. Predict the speed of sound at 60°C.



60. Answers may vary. Sample:
a. $y - 332 = \frac{3}{5}(x - 0)$
b. 341 m/s
c. 368 m/s

Challenge

Write an equation in slope-intercept form of each line described below.

61. The line contains the point $(-3, -5)$ and has the same slope as $y + 2 = 7(x + 3)$. $y = 7x + 16$

62. The line contains the point $(1, 3)$ and has the same y-intercept as $y - 5 = 2(x - 1)$. $y = 3$

63. The line contains the point $(2, -2)$ and has the same x-intercept as $y + 9 = 3(x - 4)$. $y = \frac{2}{5}x - \frac{14}{5}$

64. The table shows data that you can model using a linear function.
a. Find the value of y when $x = 6$. **14.75**
b. Find the value of y when $x = 120$. **57.5**
c. Find the value of x when $y = 11$. **−4**
d. Find the value of x when $y = 50$. **100**

x	y
4	14
8	15.5
12	17
16	18.5

Test Prep

Gridded Response

65. What is the slope of the graph of $y - 8 = \frac{1}{2}(x + 2)$? $\frac{1}{2}$

66. Find the y-intercept of the line $y + 3 = 4(x + 3)$. **9**

67. What is the x-intercept of the line $y = 3x - 7$? $\frac{7}{3}$

68. When $y - 1 = -\frac{4}{5}(x - 3)$ is written in standard form using positive integers, what is the smallest possible coefficient of x? **4**

69. When $y = -\frac{5}{2}x + \frac{2}{3}$ is written in standard form using positive integers, what is the smallest possible coefficient of y? **6**

Mixed Review

GO for Help

Lesson 6-3 **Graph each line.** **70–75. See margin.**

70. $6x + 7y = 14$ **71.** $-2x + 9y = -9$ **72.** $5x - 4y = 24$
73. $3x - 8y = 4$ **74.** $5x + 18y = 6$ **75.** $-7x + 4y = -21$

Lesson 5-7 **Find the common difference of each sequence. Then write the next two terms.**

76. $-12, -7, -2, \ldots$ **5; 3, 8** **77.** $\frac{1}{2}, \frac{5}{6}, \frac{7}{6}, \ldots$ **$\frac{2}{6}, \frac{3}{2}, \frac{11}{6}$** **78.** $2.45, 2.52, 2.59, \ldots$ **0.07; 2.66, 2.73**
79. $-3.2, -3.25, -3.3, \ldots$ **−0.05; −3.35, −3.4** **80.** $18, 35, 52, \ldots$ **17; 69, 86** **81.** $-7, -3, 1, \ldots$ **4; 5, 9**

47. $y - 1 = -\frac{1}{6}(x + 7)$; $x + 6y = -1$

48. $y - 4 = -\frac{3}{2}(x + 8)$; $3x + 2y = -16$

49. $y - 4 = 2(x - 2)$; $-2x + y = 0$

50. $y - 3 = -2(x - 5)$; $2x + y = 13$

51. $y - 1 = \frac{1}{3}x$; $-x + 3y = 3$

52. $y - 4 = -\frac{9}{2}(x + 2)$; $9x + 2y = -10$

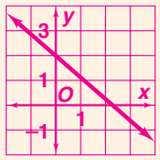
53. $y - 2 = \frac{3}{5}(x - 6)$; $-3x + 5y = -8$

Test Prep

A sheet of blank grids is available with the Test-Taking Strategies with Transparencies booklet. Give this sheet to students for practice with filling in the grids.

Resources
For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 369
- Test-Taking Strategies, p. 364
- Test-Taking Strategies with Transparencies

56a. Answers may vary. Sample: $y + 6 = 2(x + 4)$; **chose slope and substituted into** $y - y_1 = m(x - x_1)$
b. Infinitely many; any real number can be used for the slope.

70.


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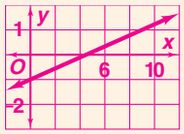

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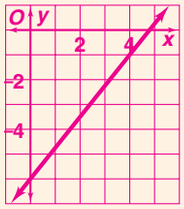

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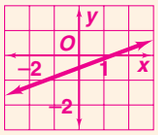

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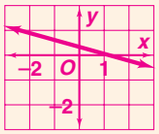
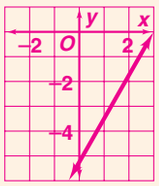
75.