What You’ll Learn
• To identify solutions of inequalities
• To graph and write inequalities

... And Why
To write inequalities for speed limits and starting salaries, as in Example 5

Vocabulary Tip
A solution of an inequality is any number that makes the inequality true. For example, the solutions of the inequality \( x < 3 \) are all numbers that are less than 3.

1. Identifying Solutions of Inequalities

1. Plan

Objectives
1. To identify solutions of inequalities
2. To graph and write inequalities

Examples
1. Identifying Solutions by Mental Math
2. Identifying Solutions by Evaluating
3. Graphing Inequalities
4. Writing an Inequality From a Graph
5. Real-World Problem Solving

Math Background
Graphing solutions on a number line helps students understand that inequalities have more than one solution.

More Math Background: p. 198C

Lesson Planning and Resources
See p. 198E for a list of the resources that support this lesson.

Bell Ringer Practice
Check Skills You’ll Need
For intervention, direct students to:
Exploring Real Numbers
Lesson 1-3: Example 4
Extra Skills and Word Problems Practice, Ch. 1

Identifying Solutions by Mental Math
Is each number a solution of \( x \leq 7 \)?

- a. 9
  - No, \( 9 \leq 7 \) is not true.

- b. \( -1 \)
  - Yes, \( -1 \leq 7 \) is true.

- c. \( \frac{14}{2} \)
  - \( \frac{14}{2} \) = 7; yes, \( \frac{14}{2} \leq 7 \) is true.

Quick Check
Is each number a solution of \( x \geq -4.1 \)?

- a. \( -5 \) no
- b. \( -4.1 \) yes
- c. \( 8 \) yes
- d. \( 0 \) yes

You can determine whether a value is a solution of an inequality by evaluating an expression.

Identifying Solutions by Evaluating
Is each number a solution of \( 2 - 5x > 13 \)?

- a. \( 3 \)
  - \( 2 - 5x > 13 \)
  - \( 2 - 5(3) > 13 \) \( \rightarrow \) Substitute for \( x \)
  - \( 2 - 15 > 13 \) \( \rightarrow \) Simplify
  - \( -13 > 13 \) \( \rightarrow \) Compare

- b. \( -4 \)
  - \( 2 - 5x > 13 \)
  - \( 2 - 5(-4) > 13 \)
  - \( 2 + 20 > 13 \)
  - \( 22 > 13 \)

3 does not make the original inequality true, so 3 is not a solution.

-4 does make the original inequality true, so \( -4 \) is a solution.

Special Needs
Have students model inequalities using a set of scales with blocks of different weights. Remind students that an equation means that the scales are in balance, while an inequality means that the scales are out of balance.

learning style: tactile

Below Level
Help students to understand that expressions such as \( -1 \geq a \) and \( a \leq -1 \) have the same meaning. Review several examples of these expressions with the students.

learning style: verbal
Graphing and Writing Inequalities in One Variable

You can use a graph to indicate all of the solutions of an inequality.

### Inequality

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &lt; 3$</td>
<td><img src="chart" alt="Graph" /></td>
</tr>
<tr>
<td>$m \geq -2$</td>
<td><img src="chart" alt="Graph" /></td>
</tr>
<tr>
<td>$-1 \geq a$</td>
<td><img src="chart" alt="Graph" /></td>
</tr>
</tbody>
</table>

The open dot shows that 3 is not a solution. Shade to the left of 3.

The closed dot shows that $-2$ is a solution. Shade to the right of $-2$.

The closed dot shows that $-1$ is a solution. Shade to the left of $-1$.

You can also write $-1 \geq a$ as $a \leq -1$.

#### Example

**Graphing Inequalities**

a. Graph $c > -2$.

b. Graph $4 \leq m$.

You can write an inequality for a graph.

#### Example

**Writing an Inequality From a Graph**

Write an inequality for each graph.

a. $x < -4$  Numbers less than $-4$ are graphed.
b. $x \leq 5$  Numbers less than or equal to 5 are graphed.
c. $x > \frac{1}{2}$  Numbers greater than $\frac{1}{2}$ are graphed.
d. $x \geq -1$  Numbers greater than or equal to $-1$ are graphed.

Choose of variable may vary.

#### Example

**Writing an Inequality From a Graph**

Write an inequality for each graph.

a. $x \geq 2$
b. $x < 0$

#### Vocabulary Tip

You normally read $-1 \geq a$ as $-1$ is greater than or equal to $a$. The inequality symbol also indicates that $a$ is less than or equal to $-1$.

#### Quick Check

2. Is each number a solution of $6x - 3 > 10$?
   a. 1 no  b. 2 no  c. 3 yes  d. 4 yes

#### Guided Instruction

1. **Math Tip**

   The inequality sign replaces the equal sign in a math sentence. The equal sign is the mathematical way of writing is. Therefore, the inequality sign is the mathematical way of writing is less than.

2. **Additional Examples**

   1. Is each number a solution of $x = 5$?
      a. $-2$ no  b. 10 yes  c. $\frac{25}{5}$ yes

   2. Is each number a solution of $3 + 2x < 8$?
      a. $-2$ yes  b. 3 no

#### Error Prevention

Tell students: Write an inequality for Kyle is older than Jaime. Kyle’s age > Jaime’s age Write this using a less than sign. Jaime’s age < Kyle’s age Lead students to understand the two inequalities have the same meaning.

#### Advanced Learners

Ask students to discuss other situations that could be represented by an inequality and determine reasonable solutions.

#### English Language Learners

Some students may be confused by the term inequality. In the word inequality, underline in. Have students discuss the meanings of inactive, incorrect, and informal. Ask: What syllable do all these words have in common? in, meaning not
Many salespeople are paid a salary plus commission. Ask students to write an inequality showing what a salesperson with a salary of $700 per week plus commission gets paid. Sample answer: $ \geq 700$

**Critical Thinking**

In part (a) of Example 5, can the speed be all real numbers less than or equal to 65? Explain. **See left above.**

In part (b) of Example 5, are all real numbers greater than or equal to $6.15$ reasonable solutions of the inequality? Explain. **See left above.**

You can describe real-world situations using an inequality.

**Real-World Problem Solving**

Define a variable and write an inequality for each situation.

(a) A job that pays at least $500 a month. Let $p = \text{pay per month (in dollars)}$.

(b) The sign indicates that $x \leq 65$.

Let $s = \text{a legal speed}$. The sign indicates that $s \leq 65$.

Let $p = \text{pay per hour (in dollars)}$. The sign indicates that $p \geq 6.15$.

**EXERCISES**

**Practice and Problem Solving**

For more exercises, see Extra Skill and Word Problem Practice.

**Mental Math**

Is each number following the inequality a solution of the given inequality?

1. $v \geq -5; 4 \text{ yes}$
2. $0.5 > c; 2 \text{ no}$
3. $b < 4; -0.5 \text{ yes}$
4. $d \leq \frac{12}{5}; 5 \text{ yes}$
5. $g \leq \frac{12}{5}; 3 \text{ no}$
6. $k < 0; -1 \text{ yes}$
7. $a > 3.2; 3 \text{ no}$
8. $x \geq -2.5; -2.5 \text{ yes}$

Is each number a solution of the given inequality?

9. $3x - 7 > -1$ a. 2 no b. 3 no c. $-1 \text{ yes}$
10. $4n - 3 \leq 5$ a. 2 yes b. 3 no c. $-1 \text{ yes}$
11. $2y + 1 < -3$ a. 0 no b. $-2 \text{ no}$ c. $1 \text{ no}$
12. $\frac{4 - m}{m} \geq 5$ a. 0.5 yes b. 2 no c. $-4 \text{ no}$
13. $mn - 3 < 54$ a. 9 no b. 3 yes c. 10 no
14. $5(2q - 8) \geq 7$ a. $-2 \text{ no}$ b. $9 \text{ no}$ c. 6 yes

Match each inequality with its graph.

15. $x < 4 \text{ C}$
16. $x \geq 4 \text{ B}$
17. $x > 4 \text{ D}$
18. $x \leq 4 \text{ A}$

A. $\cdots -3 -2 -1 0 1 2 3 4 5 \cdots$
B. $\cdots -3 -2 -1 0 1 2 3 4 5 \cdots$
C. $\cdots -3 -2 -1 0 1 2 3 4 5 \cdots$
D. $\cdots -3 -2 -1 0 1 2 3 4 5 \cdots$

19. $\cdots -1 0 1 2 3 \cdots$
20. $\cdots -5 -4 -3 -2 -1 0 \cdots$
21. $\cdots -7 -6 -5 -4 -3 -2 \cdots$
22. $\cdots -2 -1 0 1 2 \cdots$
23. $\cdots -5 -4 -3 -2 -1 0 \cdots$
24. $\cdots -3 -2 -1 0 1 \cdots$
25. $\cdots 4 5 6 7 8 9 \cdots$
26. $\cdots -1 0 1 2 3 4 5 \cdots$

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19. $x > 1$
20. $s < -3$
21. $y \leq -4$
22. $t \geq -1$
23. $-2 < d$
24. $-\frac{3}{2} \leq b$
25. $7 \geq a$
26. $4.25 > c$

Write an inequality for each graph. Choice of variable may vary.

27. $x > -3$
28. $x \leq 7$
29. $x \geq 1$
30. $x < -6$
31. $x \geq 4.5$
32. $x < -0.5$

Define a variable and write an inequality to model each situation.

33. A bus can seat at most 48 students. Let $s = \text{number of students}$. $s \leq 48$
34. In many states, you must be at least 16 years old to obtain a driver’s license. Let $a = \text{age}$. $a \geq 16$
35. It is not safe to use a light bulb of more than 60 watts in this light fixture. Let $w = \text{acceptable number of watts}$. $w \leq 60$
36. At least 350 students attended the band concert Friday night. Let $s = \text{number of students}$. $s \geq 350$
37. Aviation The Navy’s flying squad, the Blue Angels, makes more than 75 appearances each year. Let $a = \text{number of appearances}$. $a > 75$

Write each inequality in words. 38–46. See left. 47–49. See margin.

38. $n < 5$
39. $b > 0$
40. $7 \geq x$
41. $z \geq -5.6$
42. $4 > q$
43. $-1 \geq m$
44. $35 \geq w$
45. $g - 2 < 7$
46. $a \leq 3$
47. $6 + r > -2$
48. $8 \leq h$
49. $1.2 > k$
50. Writing Explain how you choose whether to draw an open or a closed dot when you graph an inequality. Use an open dot for $<$ or $>$. Use a closed dot for $\leq$ or $\geq$.
51. Open-Ended Describe a situation that you can represent using the inequality $x \geq 18$. Answers may vary. Sample: Every class has at least 18 students.
52. Multiple Choice Suppose your school plans a musical. The director’s goal is ticket sales of at least $4000$. Adult tickets are $5.00 and student tickets are $4.00. Let $a$ represent the number of adult tickets and $s$ represent the number of student tickets. Which inequality represents the director’s goal? D
   - $4a + 5s < 4000$
   - $5a + 4s \geq 4000$
   - $5a + 4s \leq 4000$
   - $5a + 4s = 4000$
Rewrite each inequality so that the variable is on the left. Then graph the solutions. 53–56. See margin for graphs.

53. $2 < x > 2$
54. $-5 \leq b - b \leq -5$
55. $0 \leq r r \leq 0$
56. $5 \geq a - 5$

Graph each inequality from the given description.

57. $t$ is nonnegative. $t \geq 0$
58. $x$ is positive. $x > 0$
59. $k$ is no more than 3. $k \leq 3$
60. $r$ is at least 2. $r \geq 2$
61. $s$ is at most 4. $s \leq 4$
62. $v$ is no less than 7. $v \geq 7$
63. Writing Explain how you interpret the phrases “at least” and “at most” in an inequality that models a real-world situation. “At least” is translated as $\geq$. “At most” is translated as $\leq$.
Lesson Quiz

1. Is each number a solution of \( x > -1? \)
   a. 3 yes
   b. -5 no

2. Is 2 a solution of \( 3x - 4 < 2? \) no

3. Graph \( x > 4. \)

4. Write an inequality for the graph. \( p \leq -2 \)

5. Graph each inequality.
   a. \( t \) is at most 2.
   b. \( w \) is at least 1.

Real-World Connection

There is an average of 2500 flights in and out of Chicago’s O’Hare Airport each day.

Air Travel

You plan to go from New York City to Los Angeles. Let \( x \) be the distance in miles of any air-route between New York City and Los Angeles. The shortest route is a direct flight. Using the map, write a true statement about the mileage of any route from New York City to Los Angeles. \( x > 2451 \)

Air Travel

Your travel agent is making plans for you to go from Chicago to New Orleans. A direct flight costs too much. Option A consists of flights from Chicago to Dallas to New Orleans. Option B consists of flights from Chicago to Orlando to New Orleans. Write an inequality comparing the mileage of these two options. Option A < Option B

Error Analysis

A student claims that the inequality \( 3x + 1 > 0 \) is always true because multiplying a number by three and then adding one to it makes the number greater than zero. Use a counterexample to show why the student is not correct. Answers may vary. Sample: For \( x = -1, 3(-1) + 1 = -3 + 1 = -2 \). -2 < 0.

Critical Thinking

Describe how you can display the solutions of the inequality \( x \neq 3 \) on a number line. Put an open dot at 3 and color the rest of the number line.

Critical Thinking

Explain the difference between “4 greater than \( x \)” and “\( 4 > x \).” “4 greater than \( x \)” means \( x + 4 \); “\( 4 > x \)” means 4 is greater than \( x \).

Critical Thinking

Which is the correct graph of \( -4 < -x \)? Explain.

A. \[
\begin{array}{cccccccc}
& & & & & & & & \\
& & & & & & & & \\
& & & & & & & & \\
& & & & & & & & \\
-3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

B. \[
\begin{array}{cccccccc}
& & & & & & & & \\
& & & & & & & & \\
& & & & & & & & \\
& & & & & & & & \\
-3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

C. \[
\begin{array}{cccccccc}
& & & & & & & & \\
& & & & & & & & \\
& & & & & & & & \\
& & & & & & & & \\
-3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

D. \[
\begin{array}{cccccccc}
& & & & & & & & \\
& & & & & & & & \\
& & & & & & & & \\
& & & & & & & & \\
-3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

\( C \); the inequality is true for \( x = 3 \) but not true for \( x = 5 \), so \( C \) is correct.

Reasoning

Give a counterexample for this statement. If \( a < b \), then \( a^2 < b^2 \).

Answers may vary. Sample: \( a = -1, b = \frac{1}{2} \)

Reasoning

Describe the numbers \( a \) and \( b \) for which the following statement is true. If \( a < b \), then \( a^2 = b^2 \); \( a \) is negative, and \( a \) and \( b \) are opposites.

Graph on a number line.

72. all values of \( x \) such that \( x > -2 \) and \( x \leq 2 \)

73. all values of \( x \) such that \( x < -1 \) or \( x > 3 \)

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Test Prep

Multiple Choice
74. Which inequality has the same solutions as \( n > 5 \)?
   A. \( n < -5 \)  
   B. \( n < 5 \)  
   C. \( 5 < n \)  
   D. \( -n > -5 \)

75. What is the least whole-number solution of \( k \geq -5 \)?
   F. \( -5 \)  
   G. \( -4 \)  
   H. \( 0 \)  
   J. \( 1 \)

76. Employees must work at least 20 years in a company in order to receive full benefits upon retirement. Which inequality or graph does NOT describe this situation?
   A. \( y \geq 20 \)  
   B. \( y > 20 \)  
   C. \( 20 \leq y \)  
   D. \( 0 \)  
   E. \( 5 \)  
   F. \( 25 \)

77. Which value makes the inequality \( x^2 \geq x \) false?
   F. \( \frac{1}{2} \)  
   G. \( 0 \)  
   H. \( \frac{1}{4} \)  
   J. \( 1 \)

78. Fire codes require that no more than 150 persons occupy a conference room. Which graph includes a room count in possible violation of the fire codes?
   A.  
   B.  
   C.  
   D.  

Short Response
79. A stretch of \( 12 \frac{1}{2} \) mi of highway is being repaired. The project foreman reported that less than half of the job is complete. Draw a diagram to show the remaining miles to be repaired. Then write an inequality for the number of miles \( m \) that still need repair. See above.

Mixed Review

Lesson 3-9
A right triangle has hypotenuse \( c \) and legs \( a \) and \( b \). Find the missing side. If necessary, round to the nearest tenth.

80. \( a = 6, b = 12 \)  
81. \( b = 5, c = 8 \)  
82. \( a = 1.2, b = 0.5 \)

83. \( c = 16, b = 7 \)  
84. \( a = 10, c = 30 \)  
85. \( a = \frac{5}{6}, b = \frac{5}{7} \)

Determine whether the given lengths can be sides of a right triangle.

86. 10, 13, 27  
87. 9, 40, 41  
88. 28, 96, 100

Lesson 2-5
Name the property that each equation demonstrates.

89. \( 3(2 \cdot 7) = (3 \cdot 2)7 \)  
90. \( 5 \times 1 = 1 \times 5 \)  
91. \( 3 + 4 = 4 + 3 \)

Lesson 1-6
Find each measure for the following data.

1 3 4 4 5 7 9 9 9 13

92. mean  
93. median  
94. mode