Multiplying Binomials

What You’ll Learn
• To multiply binomials using FOIL
• To multiply trinomials by binomials

. . . And Why
To find the area of a geometric figure, as in Example 3

Check Skills You’ll Need

Find each product.
1. \(4r(r - 1)\) \(6h^3 + 48h^2 - 18h\) \(2y^3 - 7y^2\)
2. \(4r^2 - 4r\) \(6h^2 + 8h - 3\)
3. \(3y^3 - 9y\)

Simplify. Write each answer in standard form.
4. \((x^3 + 3x^2 + x) + (5x^2 + x + 1)\)
5. \((3x^3 - 6x + 8) + (5x^3 + 7r - 2)\)
6. \(w(w + 1) + 4w(w - 7)\)
7. \(6b(b - 2) - b(b + 3)\)
8. \(m(4m^2 - 6) + 3m^2(m + 9)\)
9. \(3d^2(a^3 - 6) - d^3(2d^2 + 4)\)

Multiplying Two Binomials

You can use an area model to multiply two binomials. The diagram below shows \(2x + 3)(x + 4)\).

\[
\begin{array}{c|ccc}
& x^2 & 2x & 3 \\
\hline
x & x & x & x \\
x & x & x & x \\
2x & 2x(x + 4) & 2x^2 + 8x & 3x + 12 \\
+ & 1 & 1 & 1 \\
3 & 1 & 1 & 1 \\
\end{array}
\]

You can also use the Distributive Property to find the product of two binomials.

**EXAMPLE**

**Using the Distributive Property**

Simplify \((2x + 3)(x + 4)\).

\[
(2x + 3)(x + 4) = 2x(x + 4) + 3(x + 4) \\
= 2x^2 + 8x + 3x + 12 \\
= 2x^2 + 11x + 12
\]

**Quick Check**

Simplify each product.

a. \((6h - 7)(2h + 3)\)  
   \(12h^2 + 4h - 21\)

b. \((5m + 2)(8m - 1)\)  
   \(40m^2 + 11m - 2\)

c. \((9a - 8)(7a + 4)\)  
   \(63a^2 - 20a - 32\)

Differentiated Instruction

**Special Needs**
Have students who have difficulty in using the Distributive Property, or in drawing area models to multiply two binomials, use algebra tiles to build and explain their own solutions.

**Below Level**
Suggest that students use one method to multiply a trinomial by a binomial, and then use the other method to check their work.

learning style: tactile

learning style: verbal
One way to organize multiplying two binomials is to use FOIL, which stands for “First, Outer, Inner, Last.” The term FOIL is a memory device for applying the Distributive Property to the product of two binomials.

**Example:**

Simplify \((3x - 5)(2x + 7)\).

Set up a table to organize the multiplication using FOIL:

<table>
<thead>
<tr>
<th></th>
<th>First</th>
<th>Outer</th>
<th>Inner</th>
<th>Last</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>((3x)(2x))</td>
<td>((3x)(7))</td>
<td>((-5)(2x))</td>
<td>((-5)(7))</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{First} & = 6x^2 \\
\text{Outer} & = 21x \\
\text{Inner} & = -10x \\
\text{Last} & = -35
\end{align*}
\]

The product is \(6x^2 + 11x - 35\).

**Quick Check**

7. Simplify each product using FOIL.

a. \((3x + 4)(2x + 5)\) \(6x^2 + 23x + 20\)  
   b. \((3x - 4)(2x + 5)\) \(6x^2 + 7x - 20\)  
   c. \((3x + 4)(2x - 5)\) \(6x^2 - 7x - 20\)  
   d. \((3x - 4)(2x - 5)\) \(6x^2 - 23x + 20\)

**Example:**

**Applying Multiplication of Polynomials**

**Multiple Choice** Which expression best describes the area of the shaded region?

- A. \(x^2 + 2x\)  
- B. \(3x^2 + 17x + 5\)  
- C. \(5x^2 + 15x + 5\)  
- D. \(7x^2 + 9x + 5\)

**Example**

Find an expression for the area of each shaded region. Simplify.

a. 

\[
\begin{array}{c}
\text{area of outer rectangle} = (x + 1)(2x + 5) \\
\text{area of hole} = x(x + 2) \\
\text{area of shaded region} = \text{area of outer rectangle} - \text{area of hole} \\
= (3x + 1)(2x + 5) - x(x + 2)
\end{array}
\]

\[
\begin{align*}
&= 6x^2 + 15x + 2x + 5 - x^2 - 2x \\
&= 6x^2 - x^2 + 15x + 2x + 2x + 5 \\
&= 5x^2 + 15x + 5 \\
\end{align*}
\]

So C is the correct answer.

b. 

\[
\begin{array}{c}
\text{area of outer rectangle} = (x + 1)(x + 1) \\
\text{area of hole} = (x + 6)6x + 2 \\
\text{area of shaded region} = \text{area of outer rectangle} - \text{area of hole} \\
= (x + 1)(x + 1) - (x + 6)6x + 2 \\
= x^2 + 2x + 1 - 6x^2 - 2x - 2 \\
= -5x^2 + 1 \\
\end{array}
\]

25x^2 + 28x + 16

**Quick Check**

3. Find an expression for the area of each shaded region. Simplify.

a. \(5x + 8\)

b. \((x + 1)\)

2x - 2

**Differentiated Instruction**

**Advanced Learners**

Ask students to explain the statement, “The degree of the product of two nonzero polynomials is the sum of the degrees of the polynomials.”

**English Language Learners**

Explain that the term FOIL is a memory tool when multiplying binomials. Make sure students relate each letter to the word that identifies each part of the process (first, outer, inner, and last).
Multiplying a Trinomial and a Binomial

FOIL works when you multiply two binomials, but it is not helpful when multiplying a trinomial and a binomial. You can use the vertical method or the horizontal method to distribute each term in such factors.

**4 EXAMPLE**  Multiplying a Trinomial and a Binomial

Simplify the product \((4x^2 + x - 6)(2x - 3)\).

**Method 1** Multiply using the vertical method.

\[
\begin{array}{c}
2x - 3 \\
4x^2 + x - 6 \\
\hline
-12x^2 + 3x + 18 \\
8x^3 + 2x^2 - 12x \\
8x^3 - 10x^2 - 15x + 18
\end{array}
\]

Multiply by \(-3\).  
Multiply by \(2x\).  
Add like terms.

**Method 2** Multiply using the horizontal method.

\[
(2x - 3)(4x^2 + x - 6)
\]

\[
= 2x(4x^2) + 2x(x) + 2x(-6) - 3(4x^2) - 3(x) - 3(-6)
\]

\[
= 8x^3 + 2x^2 - 12x - 12x^2 - 3x + 18
\]

\[
= 8x^3 - 10x^2 - 15x + 18
\]

Add like terms.

The product is \(8x^3 - 10x^2 - 15x + 18\).

**Quick Check**  Simplify \((6n - 8)(2n^2 + n + 7)\) using both methods shown in Example 4.

\[
12n^3 - 10n^2 + 34n - 56
\]

**EXERCISES**

For more exercises, see *Extra Skill and Word Problem Practice.*

**Practice and Problem Solving**

Copy and fill in each blank.

1. \((5a + 2)(6a - 1) = \square a^2 + 7a - 2 \quad \square \)
2. \((3c - 7)(2c - 5) = 6c^2 - 29c + \square \)
3. \((z - 4)(2z + 1) = \square z^2 - \square z - 4 \quad \square \)
4. \((2x + 9)(x + 2) = 2x^2 + \square x + 18 \quad \square \)

Simplify each product using the Distributive Property. 5–10. See margin.

5. \((x + 2)(x + 5)\)
6. \((h + 3)(h + 4)\)
7. \((k + 7)(k - 6)\)
8. \((a - 8)(a - 9)\)
9. \((2x - 1)(x + 2)\)
10. \((2y + 5)(y - 3)\)

Simplify each product using FOIL. 11–16. See margin.

11. \((r + 6)(r - 4)\)
12. \((y + 4)(5y - 8)\)
13. \((x + 6)(x - 7)\)
14. \((m - 6)(m - 9)\)
15. \((4b - 2)(b + 3)\)
16. \((8w + 2)(w + 5)\)
17. \((x - 7)(x + 9)\)
18. \((a + 11)(a + 5)\)
19. \((p - 1)(p + 10)\)

\[
x^2 + 2x - 63
\]

\[
a^2 + 16a + 55
\]

\[
p^2 + 9p - 10
\]
3. Practice

Assignment Guide

1. A B 1-21, 30-35, 42-46
2. A B 22-29, 36-41
C Challenge 47-55

Test Prep 56-60
Mixed Review 61-86

Homework Quick Check
To check students’ understanding of key skills and concepts, go over Exercises 8, 20, 36, 39, 41.

Exercises 3, 4 Remind students to combine the “Outer” and “Inner” products to determine the middle term of the answer.

Error Prevention!
Exercises 22-25 Suggest that students write the binomial on the second line when using the vertical method.

Connection to Geometry
Exercise 43 Remind students that the bases of a trapezoid are the two parallel sides.

Example 3
Find an expression for the area of each shaded region. Simplify.

20. \(x + 3\)
21. \(x + 8\)

Example 4
Simplify. Use the vertical method. 22-25. See margin.

22. \((x + 9)(x^2 - 4x + 1)\)
23. \((a - 4)(a^2 - 2a + 1)\)
24. \((g - 3)(2g^2 + 3g + 3)\)
25. \((k + 8)(3k^2 - 5k + 7)\)

Simplify. Use the horizontal method. 26-29. See margin.

26. \((x^2 + 2x + 1)(9x - 3)\)
27. \((r^2 - 6t + 3)(2t - 5)\)
28. \((7p^2 + 5p - 1)(8p + 9)\)
29. \((12w^2 - w - 1)(4w^2 - 2)\)

Simplify each product. Write in standard form. 30-38. See margin.

30. \((p - 7)(p + 8)\)
31. \((-7 + p)(8 + p)\)
32. \((p^2 - 7)(p + 8)\)
33. \((5c - 9)(5c + 1)\)
34. \((n^2 + 3)(n + 11)\)
35. \((3k^2 + 2)(k + 5k^2)\)
36. \((6h - 1)(4h^2 + h + 3)\)
37. \((9y^2 + 2)(y^2 - y - 1)\)
38. \((8q - 4)(6q^2 + q + 1)\)

Construction
You are planning a rectangular garden. Its length is twice its width. You want a walkway 2 ft wide around the garden.

a. Write an expression for the area of the garden and walk.

b. Write an expression for the area of the walk only.

c. You have enough gravel to cover 76 ft² and want to use it all on the walk.

How big should you make the garden? 10 ft by 5 ft

Open-Ended
Write a binomial and a trinomial. Find their product. See left.

Writing
Which method do you prefer for multiplying a binomial and a trinomial? Explain. See left.

Geometry
Write an expression for the area of each shaded region. Write your answer in simplest form.

42. \(7.5x + 15\)
43. \(1.5x^2 + 25x - 1\)

44a. \(x^2 + 2x + 1, 121\)
44b. \(x^2 + 3x + 2, 132\)
44c. \(x^2 + 4x + 3, 143\)

44. a. Simplify each pair of products. a–b. See left.

i. \((x + 1)(x + 1)\)

ii. \((x + 1)(x + 2)\)

iii. \((x + 1)(x + 3)\)

b. Critical Thinking
What are the similarities between the two answers in each pair of products?

Challenge 47-55
45. **Geometry** Use the formula \( V = \ell \times w \times h \) to write a polynomial in standard form for the volume of the box. 
\( n^3 + 15n^2 + 56n \)

46. **Multiple Choice** If \( n \) represents an even number, which expression represents the product of the next two even numbers? 
D  
\( n^2 + 3n + 2 \)

47. Suppose you deposit $2000 for college in a savings account that pays 3% interest. Find the amount of money in the account after three years. 
46. 104.5 million lb  
47. 2000(1 + r)^3 \quad 48. 3000 \quad 49. 2w^2 + 7 \quad 50. a. **Vegetable Consumption** Multiply the expressions on the right side of each equation to create a model for the total number of pounds of fresh vegetables \( V(t) \) consumed in a year in the United States. 
\[ C(t) = 3.2t + 157 \quad \text{the U.S. annual per capita consumption of fresh vegetables, in pounds, from 1990 to 1997} \]
\[ P(t) = 3.3t + 250 \quad \text{the U.S. population, in millions, from 1990 to 1997} \]

b. Evaluate the equation you found in part (a) with \( t = 5 \) to find the total vegetable consumption for 1995. \((t = 0 \text{ corresponds to the year } 1990.\))

51. **Financial Planning** Suppose you deposit $2000 for college in a savings account that has an annual interest rate \( r \). At the end of three years, the value of your account will be \( 2000(1 + r)^3 \) dollars. 
\[ a. \quad 2000(1 + r)^3 + 6000(1 + r)^2 + 6000r + 2000 \]
\[ b. \quad \text{Rewrite the expression } 2000(1 + r)^3 \text{ by finding the product} \]
\[ 2000(1 + r)(1 + r)(1 + r) \]
\[ \text{Write your answer in standard form.} \]

\[ b. \quad \text{Find the amount of money in the account if the interest rate is } 3%. \quad 46. 104.5 \text{ million lb} \]

52. **Challenge** Find each product using lattice multiplication, which is explained below. 
Lattice multiplication probably originated in India in the twelfth century. It came into use in Italy in the fourteenth century. 

\[ \begin{array}{c|c|c|c}
2 & 7 \\
0 & 6 & 1 \\
6 & 1 & 9 \\
\end{array} \]

This example shows 27 \cdot 39. Each number is treated as a binomial. The four products are placed in the small, diagonally split squares. The product of 2 and 3, shown in red, is 6. The first square shows 0/6, which indicates 6. The product of 7 and 3 is 21. The second square shows 2/1.

The products are totaled diagonally. For the diagonal shaded blue, the tens place of the sum 1 + 6 + 8 is carried into the diagonal above and added into that diagonal: 1 + (2 + 6 + 1). The product 1053 appears down the left side of the lattice and across the bottom.

\[ 52. \quad 14 \cdot 72 = 1008 \quad 53. \quad 53 \cdot 87 = 4611 \]
\[ 54. \quad 91 \cdot 64 = 5824 \quad 55. \quad 38 \cdot 64 = 2432 \]

**Real-World Connection**
In 2003, the U.S. consumption of fresh tomatoes was 15.4 lb per person.

**Alternative Assessment**
Group students in pairs. Instruct one student to write a binomial and the other to write a trinomial. Then have both students multiply the pair of polynomials independently. Have students check their results with each other and discuss any discrepancies.
Multiple Choice

56. \((n - 1)(n - 4)\) is equivalent to which expression? 
A. \(n^2 - 5n + 4\)  
B. \(n^2 - 3n + 4\)  
C. \(n^2 + 3n + 4\)  
D. \(n^2 - 5n - 5\)

57. \((8k - 3)(k^2 - k + 1)\) is equivalent to which expression?  
F. \(8k^3 + 11k^2 - 11k - 3\)  
G. \(9k^3 - 8k^2 + 8k - 2\)  
H. \(8k^3 - 11k^2 - 11k + 3\)  
J. \(9k^3 - 3k^2 + 3k - 3\)

58. Which of the following products is always odd for integer values of \(n\)?  
A. \((n + 1)(n + 1)\)  
B. \((2n - 1)(2n + 1)\)  
C. \((2n - 1)(n + 1)\)  
D. \((2n + 1)(n + 1)\)

Short Response

59. Explain how to find the product of 
\((4v - 1)(2v^2 + v + 1)\), and simplify.  
See back of book.

Extended Response

60. Find an expression for the area of the shaded region. Show your work.  
See back of book.

Mixed Review

Lesson 9-2

Simplify each product. 61–68. See margin.  

61. \(4v(5v - 7)\)  
62. \((c - 9)3c\)  
63. \(8r^2(t + 6)\)  
64. \(y(3y - 10)\)  
65. \(5x^2(11 - x)\)  
66. \(-r^2(6t - 1)\)  
67. \(4r(3 - r^2)\)  
68. \(9b^2(b^3 + 2b)\)

Factor. 69–72. See margin.  

69. \(5w + 45\)  
70. \(3x^2 - 11x\)  
71. \(4a^2 + 12a\)  
72. \(9n^2 - n^3\)  
73. \(34t - 51\)  
74. \(63v^2 + 45v\)  
75. \(25m - 60m^3\)  
76. \(11k + 77k^6\)  
77. \(17(2t - 3)\)  
78. \(9v(7v + 5)\)  
79. \(5m(5 - 12m^2)\)  
80. \(11k(1 + 7k^6)\)

Lesson 8-5

Simplify each expression. 77–82.  

77. \(\frac{25}{3} - 27\)  
78. \(\frac{22}{3} - 27\)  
79. \(\frac{12}{y} \cdot y^4\)  
80. \(\frac{2w^3}{3w^3} \cdot \frac{1}{3w^2}\)  
81. \(\frac{x^8}{2x^7} \cdot \frac{1}{2x^4}\)  
82. \(\left(\frac{5}{3}\right)^{-1} \cdot \frac{3}{5}\)  
83. \(\left(\frac{3}{5}\right)^{-2} \cdot \frac{9}{25}\)  
84. \(\left(\frac{5}{3}\right)^0 1\)  
85. \(\left(\frac{3}{5}\right)^{-2} \cdot \frac{49}{16x^2}\)  
86. \(\left(\frac{3}{5}\right)^{-2} \cdot 64y^4\)

Checkpoint Quiz 1

Simplify each expression. 1–7. See margin.  

1. \((4x^2 + x + 3) + (5x^2 + 9x - 2)\)  
2. \((7b^2 - 5b + 3) - (b^2 + 8b - 6)\)  
3. \(3w(2w - 1) - 8w\)  
4. \(6k(4k + k^2) + 9k(2k - 6k^2)\)  
5. \((x + 3)(x - 5)\)  
6. \((2n^3 - 5)(6n^2 + n)\)  
7. \((g^2 + 4)(4g^2 + 8g - 9)\)  

Factor each polynomial. 8–10.  

8. \(12y^2 - 10(2y^2 - 5)\)  
9. \(5t^6 + 25t^3 - 10t\)  
10. \(18v^4 + 27v^3 + 36v^2\)  

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